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A DESCRIPTION OF THE EXPERIMENTS, INTENDED TO ILLUSTRATE A COURSE OF LECTURES, ON THE PRINCIPLES OF NATURAL PHILOSOPHY, READ IN THE OBSERVATORY AT TRINITY COLLEGE, CAMBRIDGE.

G. R. III
BRITISH MUSEUM

TO THE RIGHT REVEREND

THE BISHOP OF PETERBOROUGH,

MASTER OF TRINITY COLLEGE

IN THE UNIVERSITY OF CAMBRIDGE,

THE FOLLOWING PLAN OF A COURSE OF LECTURES

ON THE PRINCIPLES OF NATURAL PHILOSOPHY,

IS HUMBLY INSCRIBED

BY HIS LORDSHIP'S

MOST OBEDIENT,

AND MOST OBLIGED

HUMBLE SERVANT,

GEORGE ATWOOD.

London, November 12,
1776.

A
DESCRIPTION, &c.

MECHANICS.

EXPERIMENT I.

IF light bodies float upon the ſurface of water contained in a glaſs veſſel, they are obſerved to be attracted toward the ſide of it.

That power (whatever be the cauſe of it) by which bodies are impelled toward each other, has been termed by Sir I. Newton, and others, attraction, gravity, or ſometimes gravitation, which only ſignify the different modes in which the ſame principle operates. Thus the ſmaller body is ſaid to gravitate toward the larger, and the larger to attract the ſmaller.

EXP. II.

Glaſs planes, held contiguous to each other, are obſerved to attract the water in which they are immerſed, and to raiſe it up between them, above the level of the external water.

Exp. III.

Capillary tubes attract the water in which they are immerſed, and raiſe it above the level of the external water.

Exp. IV.

A copper-wire, one tenth of an inch in diameter, and three inches in length, being ſtretched with weights, is obſerved to ſuſtain 270 pounds before it is broken.

Exp. V.

Plane ſurfaces of hard bodies, being poliſhed and brought into contact, ſtrongly cohere.

Exp. VI.

Bodies by their vis inertiæ are obſerved to reſiſt the impulſes of any force tending to ſet them in motion.—Bodies in motion, reſiſt the impulſes of any force tending to augment or diminiſh their motion.

Exp. VII.

A body any how attracting another body, is attracted by it with equal force. Bodies tending toward each other, by their mutual attraction, move with velocities, which are inverſely as their quantities of matter.

The earth attracts the moon, and is equally attracted by it.— The quantity of matter in the earth is about forty times greater than that contained in the moon; whence, if theſe bodies were to approach each other in a right line by means of their mutual attraction, while the earth moved toward the moon one mile, the moon would in its approach to the earth deſcribe forty miles. And their relative velocities being always as 40 to 1 they would at length meet impinging againſt each other in contrary directions, and with equal moments.

Exp. VIII.

If a piece of gold and a feather fall at the ſame inſtant from the top of a glaſs receiver, well exhauſted of air, it is obſerved that they both deſcend to the bottom of the receiver in the ſame time.

It appears from hence that bodies are attracted toward the earth with forces not proportional to their bulks, but to the quantities of matter contained in them: for ſuppoſing the weight of the guinea to be a thouſand times greater than that of the feather, it will require 1000 times greater force to move the guinea than the feather, through the ſame ſpace in the ſame time.

Exp. IX.

If two forces act at once upon a body in the direction of the ſides of a parallelogram, and are proportional to them in quantity, it is obſerved that a force oppoſed in the direction of the diagonal, and proportional to it, will counterpoiſe the other two, and the body acted upon will remain at reſt.

Exp. X.

If a body be acted upon by three forces, repreſented in quantity and direction by the three ſides of a triangle; or by four forces, repreſented in quantity and direction by the four ſides of a trapezium, it will be at reſt.

Exp. XI.

A body, or ſyſtem of bodies, will not be at reſt unleſs their center of gravity is ſupported.

The center of gravity of an body or ſyſtem of bodies, is that point in which the weight of the whole is collected, and conſequently will deſcend to the loweſt point poſſible.

Exp. XII.

If any plane figure be placed upon a point or fulcrum, it will not reſt in any poſition whatſoever, unleſs the centers of ſuſpenſion and gravity coincide.

Exp. XIII.

If a body hang freely from a center of ſuſpenſion, it will not be at reſt, except when the line of direction produced paſſes thro' the center of ſuſpenſion.

Exp. XIV.

If a body vibrate freely round different centers of ſuſpenſion, the interſection of lines drawn from thoſe centers perpendicular to the horizon, when the body is at reſt, will be the center of gravity.

Exp. XV.

Bodies may be conſtructed in ſuch a manner, that they may appear to aſcend while their centers of gravity deſcend.

Exp. XVI.

Let two bodies, of any given magnitude and weight, be connected together by a rod, and let them revolve in an horizontal plane round different points in the rod; it is obſerved that the center of gravity will not be quieſcent, unleſs the center of motion coincides with it.

The centrifugal forces of the two bodies revolving round a point in the rod will not be equal, unleſs the centers of motion and gravity coincide.

Exp. XVII.

If a cylinder be placed upon an horizontal plane, the line of direction being without the baſe, the cylinder will fall; but if the plane is elevated in ſuch a manner, that the line of direction may fall within the baſe, the cylinder will be ſupported.

Exp. XVIII.

If ſolids of any kind be placed upon an inclined plane, when the lines of direction always fall without their baſes, they will roll down the plane; if the lines of direction fall within the baſes, and the ſurfaces beſmooth, the ſolids will ſlide down.

Exp. XIX.

Let weights be ſuſpended on a rod in the following manner: three ounces at ſeven inches from either extremity, four ounces at ten inches, and one ounce at eleven inches; then the diſtance of the center of gravity from the ſame extremity is $\frac{7\times3+4\times10+1\times11}{3+4+1}=9$ inches.

Exp. XX.

If a rod be ſo conſtructed, that a line joining the centers of gravity and ſuſpenſion may be at right angles to it, the rod hanging freely upon its center of ſuſpenſion, will not be at reſt, except in an horizontal poſition.

Exp. XXI.

A weight which counterpoiſes an ounce when ſuſpended from the longer arm of a falſe balance, being added to the weight which counterpoiſes an ounce ſuſpended from the ſhorter arm, the ſum is obſerved to be greater than two ounces.

Exp. XXII.

Let a ſmooth ſurface, loaded with weights, be placed upon an immoveable horizontal plane; if both ſurfaces be hard and well poliſhed, an horizontal force nearly equal to one third of the weight with which the upper ſurface is loaded will juſt move it along the plane.

Exp. XXIII.

Let two ſurfaces be placed upon an immoveable horizontal plane, and be loaded with equal weights; equal forces will be required to move them along the plane; but if the weights are different, the greater weight will require a greater force to move it, whatever be the proportion of the ſurfaces.

Exp. XXIV.

If a pendulum vibrate upon friction wheels, a given motion communicated to it will not be ſo ſoon deſtroyed, as when it is ſuſpended on an immoveable fulcrum.

The effects of friction may be eſtimated two ways.

1. By the quantity of motion loſt in a given time.

2. By the time in which a given quantity of motion is deſtroyed; the latter method is made uſe of in the experiment.

Exp. XXV.

There will be an equilibrium on a ſtrait lever of any kind, when the power is to the weight, as the diſtance of the weight to the diſtance of the power from the fulcrum.

For if not, let one of the weights preponderate.—Therefore the preponderateing weight has more moment than the other; but ſince by the hypotheſis, the weights are inverſely as the arms of the lever, it follows, that the velocities of the weights are inverſely as their quantities of matter; whence the moments of both are equal: but they were proved unequal. This contradictory concluſion reſults from denying the propoſition affirmed, which is therefore true. The ſame method of demonſtration may be extended to all the mechanical powers.

EXP. XXVI.

There will be an equilibrium upon a bent lever, when the power and the weight are inverſely proportional to the lines drawn from the fulcrum perpendicular to the lines of direction.

EXP. XXVII.

If weights be ſuſpended on the arms of a ſtrait lever, there will be an equilibrium, when the ſum of the products of each weight multiplied into its reſpective diſtance from the fulcrum, on one ſide, is equal to the ſum of the products taken in a ſimilar manner on the other.

EXP. XXVIII.

The ſame moment is produced on the arm of a ſtrait lever, whether weights are ſuſpended at any given diſtances from the fulcrum; or the ſum of the weights is ſuſpended at the diſtance of their common center of gravity.

Exp. XXIX.

On a fixed pulley there is an equilibrium, when the power is equal to the weight.

Exp. XXX.

On a ſingle moveable pulley, there will be an equilibrium, when the power is to the weight, as one to two.

Exp. XXXI.

There will be an equilibrium in a ſyſtem of pullies, where the ſame ſtring goes round all the pullies contained in two blocks, when the power is to the weight, as unity is to the number of ſtrings at the lower block.

Exp. XXXII.

There will be an equilibrium in a ſyſtem of moveable pullies, where a ſeparate ſtring goes round each pulley; when the power is to the weight, as unity to that power of two, whoſe index is the number of pullies.

Exp. XXXIII.

There will be an equilibrium in that ſyſtem of pullies, where the ſtring which goes

round

round each pulley is fixed to the weight; when the power is to the weight, as unity to that power of two, whoſe index is the number of pullies, the whole being decreaſed by unity.

Exp. XXXIV.

When the reſiſtance which acts againſt a wedge is perpendicular to the ſides, there will be an equilibrium, if the power be to the weight, as the back of the wedge is to the ſum of the ſides.

Exp. XXXV.

There will be an equilibrium on the ſcrew, when the power is to the weight, as the diſtance between two contiguous threads, to the circumference which the power deſcribes.

Exp. XXXVI.

There will be an equilibrium on the wheel and axle, when the power is to the weight, as the radius of the axle to the radius of the wheel.

Exp. XXXVII.

If a weight be ſuſtained upon an inclined plane, by a power which acts in a direction parallel to the plane, the power will be to the weight, as the height of the plane to the length.

Exp. XXXVIII.

If a weight be ſupported on an inclined plane, by a power which acts in a direction parallel to the baſe, the power will be to the weight, as the height of the plane to the baſe.

Exp. XXXIX.

If a weight be ſuſtained upon an inclined plane by a power which acts in a direction any how inclined to the plane, the power will be to the weight, as the ſine of the plane's elevation to the coſine of the angle, in which the direction of the power is inclined to the plane.

Exp. XL.

On a compound lever, there will be an equilibrium, when the power is to the weight, as the product of the powers on each lever, to the product of the weights.

The general rule for determining when there will be an equilibrium on a machine, compounded of any number of mechanic powers, is this: —Set down the ratios of the power to the weight on each mechanical power; the ſum of theſe will be the ratio of the power to the weight, when there is an equilibrium on the machine.

Exp. XLI.

An ivory ball impinging directly againſt a quieſcent ivory ball of equal weight, communicates to it nearly the whole velocity of the impact, and after the ſtroke remains almoſt quieſcent.

Exp. XLII.

Let the centers of any number of contiguous ivory balls, equal in weight, be placed in a right line; and let the firſt impinge againſt the next to it in the direction of the line joining the centers: It is obſerved that the ball moſt remote from the ſtroke flies off with a velocity, nearly equal to that of the ſtriking ball, which, with the intermediate balls, remains almoſt quieſcent.

Exp. XLIII.

An ivory ball impinging directly againſt a quieſcent ivory ball of twice the weight, communicates to it nearly two thirds of the velocity with which the impact is made.

Exp. XLIV.

If an elaſtic ball impinge directly againſt a quieſcent elaſtic ball of greater weight, the ſtriking ball will be reflected. If a heavier ball ſtrike againſt a lighter one at reſt, both the balls, after the impact, will go on in direction of the ſtroke.

Exp. XLV.

If two unequal weights be ſuſpended by a line joining them, and going over a fixed pulley, the heavier weight will preponderate; and in its deſcent will deſcribe ſpaces which are as the ſquares of the times of falling from reſt.

If a body fall freely by the force of gravity, it will deſcribe ſixteen feet in one ſecond, and ſixty-four feet in two ſeconds of time; The velocity of this motion, is too great for any obſervation to be made in the proportion of the times of deſcent and ſpaces deſcribed, unleſs the body falls from very great altitudes; but in this experiment, the abſolute force of gravity being diminiſhed, the velocity likewiſe will be ſo much diminiſhed, that the proportions of the times of falling, and of the ſpaces deſcribed, will be eaſily aſcertained during the deſcent through ſmaller altitudes.—If the weights be a and b, the ſpace deſcribed by the preponderating weight, will be $16t^2 \times \frac{a-b}{a+b}$ feet in t ſeconds; wherefore if $a=8\frac{1}{4}$ ounces, and $b=8$ ounces, the heavier of theſe will deſcend through about three inches in one ſecond, 3×4 inches in two ſeconds, 3×9 inches in three ſeconds, and ſo on, making allowance for the loſs of motion ariſing from friction, which muſt be very much diminiſhed in order to make the experiment nearly correſpond with the theory.—Some uſeful practical rules may be derived from the theory of bodies falling freely by the force of gravity. Suppoſe the ſpace deſcribed by a body falling from reſt in one ſecond, to be ſixteen feet, which it is nearly; (and the reſiſtance of the air will bring it ſtill nearer;) let s be any ſpace deſcribed in feet, v the velocity acquired at end of the deſcent in feet, t in the time of falling in ſeconds; and we have $s=16t^2$, $s=\frac{v^2}{64}$, $v=8\sqrt{s}$, $v=32t$, $t=\frac{\sqrt{s}}{4}$, $t=\frac{v}{32}$

Hence having given any one of theſe; viz. either the time of deſcent, the ſpace deſcribed, or velocity acquired by a body which has fallen from reſt, the other two may be readily determined.

Exp. XLVI.

If two bodies begin to deſcend from the higheſt point of an inclined plane at the ſame inſtant, one of them will fall through the perpendicular height, while the other deſcends on the plane to the interſection of the perpendicular drawn from the oppoſite angle.

Exp. XLVII.

If bodies deſcend along inclined planes which have the ſame height, the times of deſcent will be as the lengths of the planes.

For the times of deſcribing different inclined planes are directly as the lengths of the planes, and inverſely as the ſquare roots of their perpendicular heights: The heights being the ſame in this experiment, it follows that the times of deſcription will be as the lengths.

Exp. XLVIII.

If two pendelums be equal in length, it is obſerved, that they deſcribe equal arcs in equal times, whatever be the proportion of their weights.

From hence we infer that gravity acts upon bodies with forces proportional to their quantities of matter.—For ſince the two pendulums vibrate through the ſame ſpace, in the ſame time, it follows that gravity muſt act upon the heavier with a force proportionally greater than that, with which it impels the lighter.—*Vide* Exp. VIII.

Exp. XLIX.

If two pendulums be equal in length, it is obſerved that they deſcribe ſmall circular arcs in nearly the ſame time, whatever be the proportion of the arcs.

Whenever the forces which accelerate bodies are proportional to the ſpaces to be deſcribed, the times of deſcribing thoſe ſpaces will be equal. —In the preſent caſe, the forces which accelerate the pendulums are as the ſines of their angular diſtances from the loweſt point; and the arcs being ſmall, the ratio of their ſines will be nearly the ſame, as that of the arcs themſelves, or of the ſpaces to be deſcribed. Whence it follows, that the ſemi-vibrations of the pendulums moving through the different arcs, and conſequently the entire vibrations, will be deſcribed in nearly equal times.

EXP. L.

If the lengths of two pendulums be as four to one, it is obſerved that their times of vibration in the ſmall arcs of a circle are as two to one.

EXP. LI.

Let a cylindrical rod vibrate in a vertical plane on one of its extremities, as an axis of motion: a pendulum, the length of which is two thirds of the cylindrical rod, will vibrate in the ſame time.

The center of oſcillation is that point, in which all the moment of a body revolving round a center of motion is collected; in a cylindrical rod, vibrating round one of it's extremities, this point is diſtant two thirds of the whole length from the center of motion.—The diſtance of the center of oſcillation from the center of motion determines the length of a pendulum.

EXP. LII.

In a compound pendulum, as the center of ſuſpenſion is brought nearer to the center of gravity, the diſtance of the center of oſcillation from the center of ſuſpenſion increaſes ſine limite.

Let a compound pendulum conſiſt of two weights, A and B, fixed at the extremities of a rod; and let the diſtances of A and B from the center of ſuſpenſion be a and b reſpectively: If the weights be on the ſame ſide of the center of ſuſpenſion, the diſtance of the center of oſcillation from it will be $\frac{Aa^2+Bb^2}{Aa+Bb}$; if the weights be on different ſides of the center of ſuſpenſion, the diſtance of the center of oſcillation will be $\frac{Aa^2+Bb^2}{Aa-Bb}$: in this caſe when $Aa=Bb$, the center of gravity coincides with the center of ſuſpenſion, and the diſtance of the center of oſcillation from it becomes $\frac{Aa^2+Bb^2}{0}$, which is infinite.

EXP. LIII.

If the baſe of an inverted cycloid be parallel to the horizon, and bodies deſcend from different points of the arc to the loweſt point, it is obſerved that the times of deſcent are equal.

The forces which accelerate bodies deſcending on the cycloidal arc in the poſition as deſcribed in the experiment, are as the ſpaces to be deſcribed, whence the times of deſcent through them muſt be equal to each other.

EXP. LIV.

If two bodies fall from reſt at the ſame inſtant, the one deſcribing the inverted ſemicycloid, the other deſcending in a right line joining the extremities of it, the body which deſcribes the curve, will arrive at the loweſt point before that which deſcends along the right line.

The cycloid is called from this property, the line of ſwifteſt deſcent.

Exp. LV.

Bodies projected near the ſurface of the earth, in any direction not perpendicular to the horizon, deſcribe a parabola.

Experiment will not exactly agree with the theory of projectiles, unleſs the reſiſtance of the air be taken into conſideration; but the difference is not conſiderable. Some uſeful practical rules may be deduced from the principles of projectiles: In experiments made on the ſtrength of powder, and other circumſtances relating to gunnery, the angle of elevation, or the firſt direction of the project is uſually 45°, in which caſe the time of flight being given by obſervation; the horizontal range, the greateſt altitude, and the velocity of projection will be readily aſcertained.

Let $a =$ the greateſt altitude of the project
$v =$ the velocity of projection
$b =$ the horizontal range } in feet.
$t =$ the time of flight in ſeconds
then t being given we ſhall derive from it

$$a = 4t^2$$

$$b = 16t^2$$

$$v = t \times \sqrt{512}$$

Exp. LVI.

If the velocity of projection be given, the horizontal range is greateſt when the angle of elevation $= 45^\circ$.

HYDROSTATICS.

EXPERIMENT I.

LET a glaſs veſſel filled with water, be weighed in air; being afterwards weighed in water, it appears to loſe the greater part of its weight.

Before the principles of Hydroſtatics were fully underſtood, this experiment induced philoſophers to imagine, that fluids were deprived of their weight when immerſed in fluids of the ſame ſort.—The true reaſon of this apparent loſs of weight will appear in the ſucceeding experiments.

EXP. II.

Let a hollow ſphere be made heavy enough to ſink in water, and let it be weighed when immerſed: being filled with water, and again weighed when immerſed, it is obſerved to be heavier than before, by the weight of the included water.

This experiment is a direct proof, that a fluid, when immerſed in its own element, loſes no part of its weight.

EXP. III.

Let mercury be poured into tubes, open at both ends, and bent at different angles; upon immerſing the ſhorter arms in water, it is obſerved, that the mercury in all the longer arms is elevated.

One of the principal properties of fluids is their preſſure in all directions, which is demonſtrated by this experiment.

Exp. IV.

Let a piece of cork be fitted to the bottom of a veſſel, ſo that the ſurfaces may be every where in contact. If mercury be poured into the veſſel, the cork will not aſcend, till the ſurface of it is ſeperated from the bottom of the veſſel.

Cork being immerſed in mercury, is preſſed upwards with a force which is greater than its weight, and muſt therefore aſcend, unleſs the communication between its lower ſurface and the fluid is prevented: this is effected in the experiment; by which means the cork remains attached to the bottom of the veſſel, partly by its own weight, and partly by the weight of the mercury preſſing on its upper ſurface.

Exp. V.

Let oil be poured into a bent tube, open at both ends; upon immerſing the ſhorter arm in water, it is obſerved that the oil aſcends in the longer arm, and deſcends in the other.

This experiment is intended to prove, that a heavier fluid may gravitate upon a lighter.

Exp. VI.

Let a tube, open at both ends, be immerſed in mercury, contained in a glaſs veſſel; if water be poured into the veſſel, it is obſerved that the mercury will riſe in the tube.

This experiment proves, that a lighter fluid may gravitate upon a heavier.—It is alſo concluded, from hence, that however light a fluid may by be, a ſufficient quantity of it preſſing upon the ſurface of a heavier fluid, may raiſe it up an immerſed tube, with the inſide of which, the lighter fluid does not communicate.

Exp. VII.

If any number of tubes, however differing in dimenſion and figure, communicate with each other, ſo that a fluid being poured into one of them, may flow into the others, it is obſerved, that the ſurfaces of the fluid, in all the tubes, are on a level.

Exp. VIII.

Let the ſhorter arm of a ſyphon be immerſed in mercury; if the air be removed from the inſide of the tube, it is obſerved, that the mercury will flow out of the longer arm, in a continued ſtream.

It may be inferred from the preceding experiments, that the gravitation of bodies toward the center of the earth is not confined to thoſe which are uſually called heavy, but extends to matter in general; and that the terms heavy and light are merely relative expreſſions.

The relative weights of ſubſtances, or their ſpecific gravities, are meaſured by the abſolute weights contained in bodies of equal magnitudes: Thus if a cubic inch of copper, weighs nine times as much as a cubic inch of water, the relative, or ſpecific gravities of copper and water, will be as nine to one. From hence may be derived theſe general rules:

1. The weights of bodies are proportional to their magnitudes and ſpecific gravities jointly.
2. The magnitudes of bodies are as their weights directly, and ſpecific gravities inverſely.
3. The ſpecific gravities of bodies are as their weights directly, and magnitudes inverſely.

Exp. IX.

Let a circular plate of braſs be exactly fitted, ſo as to cover the lower aperture of a cylindrical tube immerſed perpendicularly in water.—If the plate be immerſed to a depth of eight times its own thickneſs, it will be juſt ſupported by the preſſure of the water.

The weight of the braſs, is equal to the weight of eight times its bulk of water; whence the preſſure on it muſt be equal to the weight of a column of water, whoſe baſe is the ſurface preſſed upon, and altitude its perpendicular diſtance from the ſurface of the water.

Exp. X.

Let a tube (open at both ends) be inſerted into a cloſed cylindrical veſſel. If water be poured in, ſo as to riſe in the tube to any altitude, it is obſerved, that the preſſure which the baſe of the veſſel ſuſtains, is equal to the weight of a cylinder of water, whoſe baſe is the ſurface preſſed upon, and altitude the ſame as that of the water in the tube.

We obſerve from this experiment, that the quantity of preſſure upon the baſe is much greater than the weight of the fluid contained in the tube and veſſel, which has been eſteemed a paradox, but will be eaſily accounted for by obſerving, that if an aperture were made in the top of the incloſed veſſel, the water by its preſſure upward, would riſe to nearly the ſame level with that in the tube; whence the water being confined by the upper part of the veſſel muſt preſs againſt the baſe with an equal force, action and re-action being equal; ſo that the preſſure ſuſtained by the baſe, conſiſts both of the weight of the fluid, and its re-action againſt the upper ſurface of the veſſel.

Exp. XI.

If a ſolid float upon the ſurface of a fluid, it is obſerved, that the fluid diſplaced is equal in weight to the ſolid.

From this experiment we may derive the following concluſion: Since the ſpecific gravities of bodies are as their weights directly, and magnitudes inverſely; if their weights are equal, the ſpecific gravities muſt be inverſely as their magnitudes.—The weight of the water diſplaced, and of the ſolid, are equal by the experiment; wherefore the magnitude of the water diſplaced, muſt be to that of the ſolid, as the ſpecific gravity of the ſolid, to the ſpecific gravity of the fluid; or, in other words, the part immerſed is to the whole, as the ſpecific gravity of the ſolid, to the ſpecific gravity of the fluid. Hence it appears, that ſhips diſplace a quantity of water equal in weight to that of the ſhip and its loading—if a quantity of water be let into the cavity of the ſhip, ſo as to make the weight of the whole, greater than the weight of an equal bulk of water, the ſhip will ſink.

Exp. XII.

If a cylinder, ſeven inches in length, the ſpecific gravity of which is nine times greater than that of water be immerſed in mercury; 2.5 inches of it will remain above the ſurface.

From the laſt experiment, the part immerſed, is to the whole as the ſpecific gravity of the ſolid to the ſpecific gravity of the fluid; or in the preſent caſe, as the part immerſed : 7 :: 9 : 14 Wherefore the part immerſed $=\frac{9\times7}{14}=4.5$: And the part remaining above the ſurface $=2.5$ inches.

Exp. XIII.

If a cylinder of elm, (or any other ſubſtance of the ſame ſpecific gravity) ten inches inlength, be immerſed perpendicularly in water, the part remaining above the ſurface is obſerved to be four inches.

2. If the ſame cylinder be immerſed in ſpirit of wine, the part remaining above the ſurface is only three inches.

Upon this principle the hydrometer is conſtructed.—For the ſpecific gravity and magnitude of the ſolid, continuing the ſame, we ſhall have the ſpecific gravity of the fluid inverſly as the part immerſed; if therefore the cylinder be graduated, the depth to which it ſinks when immerſed in different fluids, will ſhew their relative or ſpecific gravities: Thus the depth to which it ſunk in water, was obſerved to be ſix inches, in ſpirits of wine ſeven inches; wherefore the ſpecific gravity of water, is to that of ſpirits of wine, in an inverſe proportion of thoſe numbers; or, as 7 to 6, or as 1 to .857.

Exp. XIV.

Let a cylinder, ſeven inches in length, the ſpecific gravity of which is nine times greater than that of water, be immerſed in mercury; 2.5 inches of the cylinder will remain above the ſurface. If water be poured on the mercury till it covers the cylinder, 2.7 inches will now remain above the ſurface.

It followed from a preceding experiment, that if a ſolid floated upon the ſurface of a fluid, the part immerſed would be to the whole, as the ſpecific gravity of the ſolid to the ſpecific gravity of the fluid: and upon this principle, the part of the cylinder which remained above the ſurface of the mercury, was at firſt 2.5 inches; but by pouring on water, the cylinder was raiſed, ſo that 2.7 inches remained above the ſurface.—Hence the prior rule for finding the proportion of the part of a ſolid immerſed to the whole, will be accurately true only in vacuo. The rule for determining this proportion, in the preſent caſe, is as follows: as the part immerſed is to the whole, ſo is the difference of the ſpecific gravities of the ſolid and lighter fluid, to the difference of the ſpecific gravities of the heavier and lighter fluids.

It appears from hence, that if any ſubſtance float upon the ſurface of a fluid in vacuo, upon admitting the air, the floating body will riſe higher above the ſurface, ſo that the proportion of the part immerſed to the whole will be ſomewhat leſs than before.—The difference of the parts of a ſolid immerſed in a fluid, when in vacuo and in open air, may be eſtimated in general thus:

Let m = the magnitude of the ſolid
s = its ſpecific gravity
A = the part immerſed when in open air
B = the part immerſed when in vacuo
a = the ſpecific gravity of the fluid in which the ſolid is immerſed.
g = the ſpecific gravity of the air.

Then by the firſt rule $B . m :: s : a$; and $B = \frac{m \times s}{a}$ = to the part immerſed when in vacuo.

By the latter rule $A : m :: s-g : a-g$; whence $A = m \times \overline{\frac{s-g}{a-g}}$ = to the part immerſed when in air: and the difference of the parts immerſed occaſioned by the air's preſſure $= B - A = \frac{m \times s}{a} - \frac{m \times \overline{s-g}}{a-g} =$

$$\frac{m \times \overline{sa - sg - sa + ag}}{a^2 - ag} = \frac{mg \times \overline{a-s}}{a^2} \text{ nearly.}$$

Hence we may infer by making the computation, that the weight of the air incumbent on the ſurface of water, whereon a body floats, makes ſo little difference in the proportion of the part immerſed to the whole, that it may be neglected in any eſtimate of this kind, except where the utmoſt accuracy is required.

Exp. XV.

Exp. XV.

A ſolid is obſerved to weigh leſs when immerſed in Water, than in open Air, by the weight of a quantity of water equal in bulk to the ſolid.

All ſolids deſcend in a fluid with forces leſs than their weights; and the difference appears by the experiment, to be the weight of a quantity of the fluid, equal in bulk to the ſolid. By means of this principle, the relative weights or ſpecific gravities of bodies are determined. The rule is this: Let the ſolid be weighed in air and in any fluid.—Then as the whole weight is to the difference of the weights, ſo is the ſpecific gravity of the ſolid to the ſpecific gravity of the fluid.

Exp. XVI.

A cubic inch of any ſubſtance which will ſink in Water, being immerſed in it, weighs leſs by 253.2 grains than when it is weighed in air.

From this experiment is deduced a concluſion, which greatly facilitates the computation of the ſpecific gravities, magnitudes, and weights of bodies: The weight which the cubic inch loſes, in water, appears from the preceding experiment to be the weight of a cubic inch of water, which is therefore $=253.2$ grains: conſequently a cubic foot of water or 1728 inches $=253.2\times1728=437500$ grains nearly, or 1000 ounces. Let $w=$ the weight of a body in ounces; s its ſpecific gravity, taken in that ſcale, where the ſpecific gravity of water $=1000$; m its magnitude, in parts of a cubic foot; and we have the following proportion:

$$1000 :: w :: 1000\times1 :: m\times s$$

or, as the number of ounces contained in a cubic foot of water, is to the number of ounces contained in the ſolid, ſo is the ſpecific gravity of the cubic foot of water $\times$ into its magnitude, to the product of the ſpecific gravity and magnitude of the ſolid: whence $w=\frac{1000\times ms}{1000}=ms$.

Example. To find the weight of 2.16 cubic inches of braſs. Here $m=\frac{2.16}{1728}$ parts of a cubic foot: $s=8000$: and $w=m\times s=\frac{2.16\times8000}{1728}$ $=10$ ounces.

Example 2. To find the magnitude of a quantity of air, which weighs an ounce:

Here $w=1$; $s=\frac{1000}{860}$; and $m=\frac{w}{s}=\frac{860}{1000}=.86$ parts of a cubic foot, the magnitude required.

Exp. XVII.

If a very thin veſſel be weighed when full of air, and when exhauſted, the difference of weights is obſerved to be nearly $\frac{2}{7}$ of a grain for every cubic inch contained in the veſſel.

Exp. XVIII.

Let a piece of cork be exactly counterpoiſed in a ballance by a leaden weight: if the whole be placed under the receiver of an air pump, upon exhauſting the air, the cork will preponderate.

Since all bodies deſcend in vacuo with forces which are proportional to their quantities of matter, or their weights, it follows that two bodies which counterpoiſe each other in vacuo, contain accurately equal quantities of matter; but when a body deſcends in a fluid of any kind, (not tenacious) its force of deſcent is diminiſhed by the weight of a quantity of fluid equal in magnitude to the ſolid. Hence it follows that the air's preſſure diminiſhes the weight of the cork more than it does that of the lead; whence theſe ſubſtances, being counterpoiſed in air, will not contain equal quantities of matter; and the air being removed, the weight which is really the greateſt muſt preponderate.

We learn from hence that the true weights of bodies are not obtained by weighing in air, unleſs the body to be weighed, and the weight, are of the ſame ſpecific gravity.

Exp. XIX.

If a piece of copper be exactly counterpoiſed in air by a braſs weight, and both the copper and braſs be immerſed in water, the copper will preponderate.

The principle upon which this experiment depends, is this: the water preſſes upward with greater force againſt that body, which is the greateſt in magnitude; whence the braſs will loſe more weight than the copper, which muſt therefore preponderate. Let any two bodies of equal weight, but different ſpecific gravities, be immerſed in a lighter fluid; the following proportion, derived from the principles of ſpecific gravities, will determine how much greater weight is loſt by the one than by the other: as the product of the ſpecific gravities, is to their difference, ſo is the weight of either ſolid × into the ſpecific gravity of the fluid, in which the ſolids are immerced, to the difference of weights required.

Example. If 120 grains of braſs be counterpoiſed in air by a copper weight, upon immerſing them in water, the equilibrium is deſtroyed: the weight which muſt be added to the ſcale from which the braſs is ſuſpended, in order to reſtore the equilibrium, will be determined by referring to the rule:

As $9 \times 8 : 9-8 :: 120 \times 1 : \frac{120}{72} =$ one grain and two thirds, the weight required.

Example 2. If a guinea weighing 129 grains be counterpoiſed in air by a braſs weight, and both be immerſed in water, $\frac{129 \times \overline{17.2-8}}{8 \times 17.2}$ ~~[illegible]~~ or ſomething more than $8\frac{1}{2}$ grains muſt be added to the ſcale from which the braſs is ſuſpended in order to reſtore the equilibrium which is deſtroyed by the immerſion.

Exp. XX.

The ſpecific gravity of a body which will not ſink in water may be determined by uniting it to a ſolid, ſo that the compound may be ſpecifically heavier than water.

This method of determining ſpecific gravities may be expreſſed generally thus:

Let A = the weight of the heavier ſolid
a = its ſpecific gravity
B = the weight of the lighter ſolid
x = its ſpecific gravity ſought
C = the weight of the compound
c = its ſpecific gravity.

Then from the principles of ſpecific gravities, we have $\frac{A}{a} + \frac{B}{x} = \frac{C}{c}$ whence $x = \frac{Bac}{Ca - Ac}$, the ſpecific gravity required.

Exp. XXI.

Let a glaſs tube, 36 inches in length, be filled with mercury, and one extremity being cloſed, let the other be immerſed in a veſſel of mercury; it is obſerved, that the mercury ſinks from the upper end of the tube, and remains ſuſpended at an altitude of about twenty-nine inches and a half.

Exp. XXII.

A tube leſs than twenty-eight inches in length, being filled and immerſed, the the mercury will remain contiguous to the upper end of the tube.

The weight of the mercury in the tube is this caſe leſs than the preſſure of the atmoſphere.

Exp. XXIII.

If barometer tubes, of various dimenſions and inclinations, be filled and immerſed, it is obſerved, that the mercury remains ſuſpended at the ſame perpendicular altitude in them all.

Exp. XXIV.

Let a barometer be placed under the receiver of an air-pump, as the air is exhauſting, the mercury continually deſcends 'till it is nearly on a level with the mercury in the veſſel.—Upon admitting the air, the mercury riſes to its former altitude.

Exp. XXV.

Let a tube, thirty-ſix inches in length, open at both ends, be immerſed in mercury, contained in a veſſel, from the inſide of which the external air is excluded; if the tube and veſſel be placed under a receiver, it is obſerved, upon exhauſting the air, that the mercury is raiſed in the tube nearly to the ſtandard altitude.

It is demonſtrated from this experiment, that the elaſtic force of the air preſſing againſt any ſurface, is equal to the weight of the atmoſphere preſſing upon the ſame ſurface.

Exp. XXVI.

Let a veſſel be filled with water, and the aperture of it be exactly covered with a braſs plate; if the veſſel be inverted, the plate will continue to adhere to it.

Exp. XXVII.

Let a thin glaſs veſſel, the aperture of which is cloſed, be placed under the receiver of an air-pump, if the air be exhauſted from the receiver, the veſſel will be broken.

Exp. XXVIII.

Let a thin glaſs veſſel, the aperture of which is cloſed, be fitted with a valve, opening outwards. If this be placed under the receiver of an air-pump, the air being exhauſted, and re-admitted into the receiver, the veſſel will be broken.

Exp. XXIX.

Let a thin plate of glaſs be fitted to the upper aperture of a receiver, ſo that there be no communication between the internal and external air; upon exhauſting the receiver, the plate of glaſs is broken.

Exp. XXX.

If a barometer be immerſed in a veſſel of mercury, and ſuſpended from the beam of a ballance; it is obſerved that the weight neceſſary to counterpoiſe it, excluſive of the tube, is equal to the weight of the mercury ſuſtained in the barometer, by the preſſure of the atmoſphere.

Exp. XXXI.

Let two hollow braſs hemiſpheres be cloſely placed together ſo as to prevent the communication of the external and internal air; if the air be exhauſted from the cavity, it is obſerved that the hemiſpheres are preſſed together with a force which requires a weight of about fifteen pounds for every ſquare inch of a great circle of the biſſected ſphere to ſeparate them.

Exp. XXXII.

If the exhauſted hemiſpheres be ſuſpended under the receiver of an air-pump, upon exhauſting the air, the lower hemiſphere will fall to the bottom of the receiver.

Exp. XXXIII.

If an exhauſted receiver be made to communicate with a veſſel of water, by means of a tube inſerted in the center of the plate on which the receiver ſtands; the water aſcends through the tube, and ſtrikes againſt the extremity of the receiver with a continued ſtream.

Exp. XXXIV.

Let a glaſs ſphere, having a ſmall orifice made in that part of it which is immerſed, float upon the ſurface of a veſſel of water: if the veſſel be placed under the receiver of an air-pump, upon exhauſting the air, and re-admitting it into the receiver, the ſphere will be filled with water, and ſink to the bottom of the veſſel.

Exp. XXXV.

Warm water, or newly fermented liquors, appear to boil, in vacuo.

Exp. XXXVI.

The ſpaces into which a given quantity of air is compreſſed, are obſerved to be leſs in the ſame proportion, as the forces of compreſſion are greater.

Exp. XXXVII.

Let ten inches of air be included with the mercury in a barometer (exactly cylindrical) of thirty-five inches in length; if the barometer be inverted and immerſed in mercury, the included air will now occupy twenty inches.

From this experiment is derived the following proportion: As the ſtandard altitude is to the defect from it, ſo is the ſpace which the included air occupies after inverſion, to the ſpace it occupied before inverſion.

Let $s =$ the ſtandard altitude
$d =$ the defect from it
$b =$ the difference between the ſtandard altitude and the length of the tube
$g =$ the quantity of air included.

Whence we have in general $s : d :: b + d : g$ and $g = \overline{b+d} \times \frac{d}{s}$

If the quantity of air included be given to find the defect from the ſtandard altitude, put x equal to the defect, and we have by the preceding rule:

$s : x :: b + x : g$. Wherefore $x^2 + bx = sg$; and $x = \frac{\sqrt{b^2 + 4sg} - b}{2}$.

If the ſtandard altitude be greater than the length of the tube, the ſign of b muſt be changed, and the defect from the ſtandard altitude, or x will now be equal to $\frac{\sqrt{b^2 + 4sg} + b}{2}$

The barometer tube being cylindrical, the quantity of air included before inverſion, and the ſpaces occupied after inverſion are eſtimated by the degrees, into which the length of the tube is divided.

A general concluſion is derived from the two preceeding experiments, viz. the elaſtic force of the air is directly proportional to it's denſity, and inverſely as the ſpace which it occupies.

Exp. XXXVIII.

The altitude of the mercury in a barometer, elevated above the ſurface of the earth, is obſerved to be leſs than the ſtandard altitude.

Since the ſuſpenſion of the mercury in the barometer, has been proved to ariſe from the preſſure of the atmoſphere, it follows, that if any part of the air's weight ſhould be taken off, the altitude of the mercury in the tube will be diminiſhed.—The following rule will determine the height of a mountain by means of the barometer: (ſetting aſide the effects of the different temperatures of the air.)—Let the altitude of the mercury in the barometer at the bottom of the mountain be a, the altitude at the top b: the height required will be $= \log. \frac{a}{b} \Big| \times \frac{70}{6}$ in parts of a mile.

The log. of $\frac{a}{b}$ is here ſuppoſed to be *Briggs*'s.

Exp. XXXIX.

If a tube be immerſed in a fluid, and the air contained in the tube be rarefied, the fluid will riſe in it.

This experiment is ſimilar to Exp. XXV. but is immediately deſigned to explain the operation of pumps in raiſing water : theſe inſtruments, however various in their conſtruction, all depend upon the ſame principles, viz. the weight and elaſticity of the air.

Exp. XL.

If a tube, open at both ends, be immerſed in water, the ſurface of which is contiguous to air, compreſſed into one half the ſpace, which it poſſeſſes in the atmoſphere, the water will ſpout from the tube when placed vertically, to an altitude of about thirty-three feet, ſetting aſide the effects of friction, and the reſiſtance the air.

Exp. XLI.

If a bell be ſtruck under the receiver of an air-pump, it is obſerved that the ſound becomes weaker the more the air is exhauſted.

It appears from this experiment, that ſound is propagated by the air ; but on account of the parts of the apparatus which communicate with the bell, ſome ſound will always be perceived, though the air is as much exhauſted as poſſible.

Exp. XLII.

Flame is obſerved to be extinguiſhed in vacuo. If it be placed under a cloſe receiver, it will be extinguiſhed after ſome time, although the air be not exhauſted.

Exp. XLIII.

If water flows out of equal apertures made in the ſides of cylindrical veſſels, and the veſſels be kept conſtantly filled, it is obſerved that the quantity of water diſcharged in a given time is in a direct ſubduplicate ratio of the perpendicular diſtance of the apertures from the ſurface of the water.

Exp. XLIV.

If an aperture biſſects the altitude of a cylindrical veſſel, kept conſtantly filled with water, the horizontal diſtance to which the water ſpouts, is equal to the height of the veſſel, ſetting aſide the effects of friction, and the reſiſtance of the air.

Exp. XLV.

If apertures be made at different diſtances from the ſurface of the water contained in a cylindrical veſſel kept conſtantly filled, the horizontal diſtance to which the fluid ſpouts, is obſerved to be the greateſt when the aperture biſſects the diſtance between the ſurface of the water and baſe of the veſſel.

In general, let A be the altitude of the veſſel, D the diſtance of the aperture from the ſurface, then the parameter of the parabola deſcribed by the fluid, will be $4D$, the abſciſſa $= A - D$: And the ſquare of the ordinate, or of the horizontal diſtance is equal to $4AD - 4D^2$, which quantity is the greateſt when $A = 2D$, or when the aperture biſſects the diſtance between the ſurface of the water, and the baſe of the veſſel.

Exp. XLVI.

Let apertures be made at equal diſtances from the bottom, and the top of a cylindrical veſſel kept conſtantly filled with water; the horizontal diſtances to which the water ſpouts from theſe apertures is obſerved to be the ſame.

ELECTRICITY.

EXPEIMENT I.

IF a glaſs tube made very dry, be rubbed with a piece of ſilk, it is obſerved that light ſubſtances, ſuch as feathers, cork, balls, &c. being applied near to it, are firſt attracted and afterwards repelled.

The power by which theſe light ſubſtances are attracted and repelled, is called Electricity; the ſubſtance by the friction of which it is produced, is ſaid to be electric; the friction of electric bodies, is called excitation.

EXP. II.

Excited ſealing-wax being applied near to light bodies, firſt attracts and afterwards repels them.

Amber, ſilk, jet, dry wood, and a variety of other ſubſtances being excited, attract and repel light bodies, and are called Electrics.—Metals, water and other bodies, the friction of which will not produce this power of attraction, and repulſion, are called Non-electrics.

EXP. III.

If a metalline cylinder be ſupported upon ſilk lines, or upon glaſs, and an excited electric be applied near to it, every part of the cylinder will attract and repel light ſubſtances, in the ſame manner as the excited electric itſelf.

Metals poſſeſſing the power of tranſmitting electricity, are called Conductors; to theſe may be added water, and all bodies which contain it.

EXP.

Exp. IV.

Let a ſilk line, or a glaſs rod made very dry, be ſuſpended on glaſs; it is obſerved that electricity cannot be tranſmitted through either of them.

Glaſs being impervious to electricity is called a Non-conductor. Silk, jet, ſealing-wax, air, &c. are likewiſe Non-conductors; and in general, all bodies which are electrics, are Non-conductors; and all bodies which are Non-electrics, are conductors.

A body which communicates with nothing but electrics, is ſaid to be inſulated.

Exp. V.

Let two cork balls be ſuſpended from lines about ſix inches in length, and inſulated; if excited ſealing-wax or glaſs be applied to them, they will repel each other; and after the excited electric is removed, they are obſerved to remain repelled.

The uſe of inſulating bodies, is to confine the electricity applied to them, and to render the effects of it permanent.

Exp. VI.

Conducting ſubſtances not inſulated, being held near to an excited electric, are obſerved to be attracted towards it.

Definition. — The electric power which is produced from the excitation of glaſs, is called the Vitreous Electricity; and the power which is produced from the excitation of ſealing-wax is called the Reſinous Electricity.

Exp. VII.

If the inſulated cork balls repel each other with the vitreous electricity, the reſinous power being applied to them, will deſtroy that repulſion, and will cauſe the balls to repel each other with the reſinous electricity, if the excitation be ſufficiently ſtrong.

EXP. VIII.

The repulſion of the balls electrified with the reſinous power, is deſtroyed by the application of the vitreous.

EXP. IX.

If glaſs and ſealing-wax equally excited, are at once applied to the inſulated cork balls, no electricity of either ſort will appear to be communicated to them.

We obſerve from the three preceding experiments, that the vitreous and reſinous powers counteract each other; whence if both are applied at once to a body, the electricity communicated upon the whole, will be only the difference of the two, and will be of that ſort which is the ſtrongeſt.

EXP. X.

If bodies be electrified with contrary powers, they are obſerved to attract each other ſtrongly.

General Properties of Electricity.

1. Bodies electrified vitreouſly repel each other.
2. Bodies electrified reſinouſly repel each other.
3. Electrified bodies attract bodies not electrified.
4. Bodies electrified by contrary powers attract each other ſtrongly.

Exp. XI.

If a glaſs tube having it's poliſh taken off be excited, it will communicate the reſinous electricity to bodies held near it.

Exp. XII.

A piece of down being applied to excited ſealing-wax, and touched at the ſame time with a conductor, is electrified with the vitreous power.

It is concluded from this and the preceding experiment, that both reſinous and vitreous powers are produced from the excitation either of glaſs or of ſealing wax.—For the ſake of diſtinction, that power which is produced by the excitation of poliſhed glaſs, will be called vitreous, and the contrary power reſinous.

Exp. XIII.

If the rubber of a glaſs tube or cylinder be inſulated, and a conductor applied to rhe cylinder while excited, it is obſerved that the rubber is electrified reſinouſly.

Whenever the electric powers are ſeparated by excitation, one power attaches itſelf to the excited electric, and the other to the rubber.

Exp. XIV.

If the rubber and conductor applied to an excited electric, are both inſulated, the leſs electricity will be produced, the more perfect the inſulation is made.

Neither the rubber nor conductor can be inſulated perfectly, on accoun of the moiſture which is always floating in the air.

If the air and the parts of the apparatus be very dry, little or no electricity will be excited in the circumſtances above-mentioned.

It is concluded from hence that the electric powers do not exiſt in the electrics themſelves, but that they are produced from the earth by the excitation of electries.

An electric ſubſtance contained between paralled ſurfaces, however diſpoſed, is called an electric plate.

Exp. XV.

Electrified bodies attract bodies not electrified, although a thin electric plate be interpoſed between them.

Exp. XVI.

Bodies electrified with contrary powers, attract each other ſtrongly, although an electric plate be interpoſed between them.

Thus flame communicates heat through ſubſtances, which are impervious to the flame itſelf.

HYPOTHESIS.

1. The two electric powers exiſt together in all bodies.
2. Since they counteract each other when united, they can be made evident to the ſenſes, only by their ſeparation.
3. The two powers are ſeparated in nonelectrics by the excitation of electrics, or by the application of excited electrics.
4. The powers cannot be ſeparated in electric ſubſtances.
5. The two electricities attract each other ſtrongly through the ſubſtance of electrics.
6. Electric ſubſtances are impervious to the two electricities.
7. Either power, when applied to an unelectrified body repels the power of the ſame ſort, and attracts the contrary power.

Exp. XVII.

If one ſurface of a plate be electrified with either power, the oppoſite ſurface is electrified with the contrary power, if it be not inſulated.

Either power being applied to one of the ſurfaces, attracts the contrary power through the ſubſtance of the electric, and repels the electricity of the ſame ſort with itſelf.—The two powers being thus brought to the oppoſite ſurfaces of the plate which is impervious to them, remain ſuſpended; ſtrongly attracting each other, till the interpoſed plate is broken by their force, or a communication is formed between the two ſurfaces by ſome conducting ſubſtance.

When the two powers are ſuſpended on the oppoſite ſurfaces of an electric plate; the plate is ſaid to be charged. A conducting ſubſtance which forms a communication between two charged ſurfaces is called a circuit.

Exp. XVIII.

Whenever two powers unite by breaking a plate of air, an electric light is obſerved, which is attended with an exploſion if the charge be conſiderable; the union of the two electricities deſtroys the effects of each, and leaves the plate diſcharged.

Whenever either power is accumulated in an electric cloud, the ſurface of the earth oppoſed to it is electrified by the contrary power.—Theſe powers ſometimes unite by breaking the interpoſed plate of air.

Exp. XIX.

If plates of air and glaſs equal in magnitude and dimenſion, are charged by equal excitations, the plate of air is obſerved to be much more eaſily broken by the force of the charge, than the plate of glaſs.

Exp. XX.

If inſulated bodies be applied to either ſurface of a charged plate, at different diſtances from it, it is obſerved that the leſs the diſtance, the ſtronger the repulſive power. If bodies not inſulated are applied, the leſs the diſtance from the charged ſurface, the ſtronger is the power of attraction; from whence it is concluded, that the forces of electric repulſion and attraction vary in ſome inverſe ratio of the diſtance.

It appears that every thing elſe being the ſame, a thin electric plate of glaſs will explode with greater force, than a thicker plate, the attraction of the two powers being the ſtronger, the nearer the charged ſurfaces are to each other.

Exp. XXI.

The force of an electric charge is obſerved not to depend upon the ſhape in which the charged ſurfaces of a given magnitude and thickneſs are diſpoſed.

It may be inferred from the preceeding experiments, that the force of a charged plate depends upon two things:

1. The quantity of the ſurfaces charged.
2. Their proximity.

When the ſurfaces of an electric plate are charged with a certain quantity of the two powers, it is obſerved that no additional electricity can be communicated to them, however great an excitation is applied for that purpoſe.

Exp. XXII.

If any part of the human body forms a portion of the electric circuit, it is obſerved that the diſcharge is perceived in that part only which forms the communication, unleſs the charged ſurfaces are very great.

Exp. XXIII.

If circuits different in length and of different ſubſtances form a communication between two charged ſurfaces of an electric plate, it is obſerved that the diſcharge will be made through the beſt conductor, whatever be the lengths of the others.

2. If circuits of the ſame ſubſtance be different in length, the diſcharge will be made through the ſhorteſt of them.

3. If the circuits be the ſame in every reſpect, the diſcharge will be made through many of them at the ſame time.

EXP. XXIV.

The times of diſcharging the ſurfaces of an electric plate through the longeſt or ſhorteſt circuits are not ſenſibly different.

Many miles of the earth's ſurface, large tracts of water, &c. have been made a part of the electric circuit, and the diſcharge has been inſtantaneous through them all.

EXP. XXV.

If either ſurface of a charged plate communicate with the earth, the power on the oppoſite ſurface will expand itſelf into a conductor which is contiguous to it, although the conductor be inſulated.

EXP. XXVI.

Let one ſurface of a charged plate be inſulated; although the other communicate with the earth, no diſcharge of either ſurface will follow.

Exp. XXVII.

Let the ſurfaces of an electric plate be very ſlightly charged and inſulated, and let an interrupted circuit be formed; the two powers will be viſible, illuminating the points of the interrupted circuit, and each power will appear to extend farther from the ſurface contiguous to it, the ſtronger charge is communicated to the plate; but if the illuminations on each ſide meet, there will immediately follow an exploſion of the whole charge.

A cylindrical plate is that which is contained between two equal circles, the planes of which are parrallel to each other.

Exp. XXVIII.

If a cylindrical plate of air contained in the receiver of an air-pump be charged, it is obſerved that the more air is exhauſted from between the ſurfaces, the more eaſily the two powers will unite.

The diſcharge appears to be made from that ſurface which poſſeſſes the vitreous power.

Exp. XXIX.

If an exhauſted receiver be made part of the electric circuit, and the charge ſhould be not ſufficient to cauſe an exploſion, an electric light will appear to proceed in oppoſite directions from the parts communicating with the vitreous and reſinous ſurfaces.

If two contiguous clouds, very high in the atmoſphere, be electriſied with contrary powers, their diſcharge cauſes an appearance ſimilar to the diffuſe light obſerved in the receiver.

Exp. XXX.

The ſtrength of the excitation being the ſame, it is obſerved that a charged plate of air will be the more eaſily broken, the ſmaller quantity of air is interpoſed between the oppoſite ſurfaces.

Hence an electric plate is more eaſily diſcharged through an exhauſted receiver than in the open air, for the more rare the air is made by exhauſtion, the leſs reſiſtance will be oppoſed to the union of the two powers poſſeſſing the oppoſite ſurfaces, and attracting each other through the electric.

PROPOSITION.

The leaſt cylinder, the diameter of which is finite, and the thickneſs evaneſcent, is to the leaſt cylinder, the thickneſs of which is finite, and the diameter evaneſcent, in a greater than any aſſignable ratio.

Exp. XXXI.

The electric powers poſſeſſing the oppoſite ſurfaces of a cylindrical plate of air, whoſe diameter is evaneſcent, will be incomparably leſs reſiſted by the interpoſed electric, than by the thinneſt plate of air contained between two ſurfaces of a finite magnitude.

Exp. XXXII.

Let a pointed body be oppoſed to either ſurface of a charged electric plate; it is obſerved that whatever be the magnitude of the ſurfaces or the quantity of electricity, the whole will be ſilently diſcharged if the other ſurface be not inſulated.

There being no reſiſtance to the union of the two powers, there can be no accumulation or condenſation of them.

Hence the diſcharge will be made ſilently and gradually through a current formed between the point and the ſurface oppoſed to it.

Thus when an electric cloud and the ſurface of the earth directly under it are charged with contrary powers, they attract each other through the interpoſed plate of air.—It is obſerved, that whatever be the quantity of the electric powers contained in the oppoſite ſurfaces, the whole is ſilently diſcharged, if the cloud approaches a pointed body communicating with the earth.

If a conical pointed body were inſerted into ſimilar hollow cone, formed in an electrified ſolid, the ſurfaces of the two cones being every where equi-diſtant, no greater diſcharge of the electricites would follow, than if the two conical ſurfaces had been plane and oppoſed to each other at the ſame diſtance.

In order to increaſe the force of the charge, a number of electric plates are ſometimes joined together—the uſual method is, to form a communication between the inſides of a number of coated glaſs jars, and another communication between the external coating. Jars ſo diſpoſed are called a battery, the effects of which are exactly ſimilar to thoſe of a ſingle plate of the ſame thickneſs with the jars, and containing the ſame quantity of ſurface.

Exp. XXXIII.

If a large battery be diſcharged through a ſlender piece of wire, the wire will be broken to pieces or melted.

Gunpowder or ſpirits of wine will be fired, if they are a part of the circuit.—And if the diſcharge be made through a ſilver leaf confined between two glaſs planes, the glaſs will be broken to pieces and ſtained by the ſilver, which is driven by the force of the diſcharge, into the ſubſtance of the glaſs.

Exp. XXXIV.

If a battery be diſcharged through a quire of paper, a perforation is made through it, and each of the leaves is protruded by the ſtroke from the middle toward the outward leaves.

Exp. XXXV.

If two electric plates be charged, and a communication formed between the vitreous ſide of one, and the reſinous ſide of the other, no diſcharge will follow; unleſs a communication be formed between the other two ſurfaces, at the ſame time.

The natural electricity in the atmoſphere is frequently diſcharged in this manner: Two clouds being electrified with oppoſite powers, the ſurfaces of the earth immediately under them are likewiſe electrified with powers contrary to thoſe in the clouds above them; and the moiſture of earth forming a communication between the two contiguous charged ſurfaces, whenever the two clouds meet there will follow a diſcharge, both of the clouds and the ſurfaces on the earth oppoſed to them.—If the earth ſhould be dry, and conſequently afford a reſiſtance to the union of the two electricities accumulated on its ſurface, there will follow an exploſion in the earth, as well as in the atmoſphere, which will produce concuſſions and other phænomena which have frequently been obſerved to happen in dry ſeaſons, particularly in thoſe climates which are the moſt liable to ſtorms of thunder and lightning.

MAGNETISM.

EXPERIMENT I.

LET a ſlender iron rod exactly ballanced, be ſuſpended on a point ſo as to revolve freely in a plane parallel to the horizon: If the rod be touched with a loadſtone, one extremity of it will be always directed towards the north.

That power which the loadſtone communicates to iron is called Magnetiſm; a rod of iron poſſeſſed of this power is called a Magnet: Magnetiſm is communicated to iron by magnetical rods, as well as by the loadſtone itſelf: Iron rods alſo become magnetical by friction, by being kept for a long time in the ſame poſition, and by electricity, without any aſſiſtance from the loadſtone or magnets. The magnetical needle does not point exactly to the north, but is obſerved to change it's azimuth; ſometimes pointing toward the eaſt, and ſometimes to the weſt of the meridian. The extremities of a magnet are called Poles. A vertical circle in the Heavens which interſects the horizon in the points to which the magnetical needle, when at reſt, ~~points, is~~ called the Magnetic Meridian. is directe

EXP. II.

If the north pole of a magnet be placed at the point of ſuſpenſion of a needle, and drawn along towards either extremity; that part of the needle which is touched by the magnet, will be directed to the ſouth.

EXP. III.

If the ſouth pole of a magnet be placed to the point of ſuſpenſion of a needle, and drawn along toward either extremity; that part of the needle which is touched, will be directed to the north.

Exp. IV.

Either pole of a magnet is obſerved to attract iron, which is not magnetical.

Exp. V.

The north poles of two magnets repel each other when brought contiguous.

Exp. VI.

The ſouth poles of two magnets repel each other when brought contiguous.

Exp. VII.

The contrary poles of two magnets attract each other ſtrongly.

Exp. VIII.

The northern magnetiſm is obſerved to be deſtroyed by the communication of the ſouthern and *vice verſa*.

It appears from hence, that the two ſorts of magnetiſm operate in ſuch a manner as to counteract each other; that if both be communicated to the arm of a needle, the effect upon the whole will be only the difference of the two; and will be of that ſort which is the ſtrongeſt.

Hence it is eaſy to reverſe the poles of the magnet.

Exp. IX.

Let a needle be exactly ballanced and ſuſpended, ſo as to revolve freely in a vertical plane; if it be made magnetical when horizontal, the north pole will be depreſſed, and the ſouth pole elevated above the horizon.

Exp. X.

The diſcharge of the electric powers through a needle, communicates magnetiſm to it, if the charge be ſufficient.

General Properties of Magnetiſm.

1. No ſubſtance is capable of communicating or receiving magnetiſm, except the loadſtone and iron.
2. Every magnet has two poles.
3. Similar poles of magnets repel each other.
4. Either pole attracts iron not magnetical.
5. The contrary poles of magnets attract each other ſtrongly.
6. The properties of electricity and magnetiſm, are, in ſome reſpects analogous to each other.—*Vide* Electricity, Exp. X and XVI.

OPTICS.

OPTICS.

EXPERIMENT I.

RAYS of light proceed from luminous bodies in right lines and in all directions.

It is generally agreed among philoſophers, that light conſiſts of very minute particles of matter, which are ſent forth by ſome power, the cauſe of which is unknown, from luminous bodies, in right lines and in all directions.

Single particles of light, ſucceeding each other in a right line, conſtitute a ray of light, conſidered in a mathematical ſenſe; but, phyſically ſpeaking, a ray is the leaſt viſible portion of light ſeparated from the reſt.

A ray of light being turned out of its courſe by falling upon a poliſhed ſubſtance through which it does not paſs, is ſaid to be reflected.

The angle contained between the incident ray, and a line drawn perpendicular to the ſurface, from the point whereon the ray falls, is called the angle of incidence.

The angle contained between the perpendicular and the reflected ray, is called the angle of reflection.

EXP. II.

The incident and reflected rays are in the ſame plane, and the angles of incidence and reflection are equal.

The rays do not impinge upon the reflecting ſurface, being turned out of their courſe before they arrive at it, by ſome power which is contiguous to the ſurface and acts in a direction perpendicular to it.

EXP. III.

Rays of light proceed from illumined bodies, in right lines and in all directions.

The ſurfaces of thoſe bodies which conſiſt of uneven and irregular parts, reflect the light in all directions.

A ray of light being turned out of its courſe by falling obliquely upon the ſurface of a tranſparent medium which it paſſes through, is ſaid to be refracted; the angle contained between the incident ray, and a perpendicular to the ſurface, drawn from the point whereon the ray falls, is called the angle of incidence; the angle contained between the refracted ray and the perpendicular above-mentioned is called the angle of refraction.

The difference of the angles of incidence and refraction, is the angle by which the ray deviates from its original direction, and is called the angle of deviation.

Rays of light are refracted by ſome power contiguous to the refracting ſurface.

Rays of light being bent out of their courſe, by paſſing near the ſharp edges of bodies, are ſaid to be inflected.

Exp. IV.

When a ray falls obliquely upon the ſurface of a denſer medium, it is refracted towards the perpendicular; a ray, falling obliquely upon the ſurface of a rarer medium, and paſſing through it, is refracted from the perpendicular.

A ray, falling perpendicularly upon a refracting ſurface, is not turned out of its courſe, but proceeds in the ſame direction with the incident ray.—While a ray moves in the ſame uniform medium, it does not change its direction.

The principles of Optics are demonſtrated upon a ſuppoſition that light is an homogeneal ſubſtance; and though light will appear to be compounded of ſeveral kinds of rays, yet the principles of refraction and reflexion, &c. are mathematically true when [illegible]ed to rays of any one ſort,

Exp. V.

When a ray of light is refracted out of air into a given medium, or out of a given medium into air, the ſine of the angle of incidence is to the ſine of the angle of the refraction in a given ratio.

If a ray of light is refracted out of air into glaſs, the ſine of incidence is to the ſine of refraction as 31 : 20, or as 3 to 2 nearly; out of air into water, as 4 to 3. Hence the ſine of incidence is to the ſine of refraction, when a ray paſſes out of water into glaſs, as $\frac{3}{4} : \frac{20}{31}$, or as 93 : 80.

Exp. VI.

A ray of light cannot paſs out of a denſer medium into a rarer, if the angle of incidence exceeds a certain limit.

This limit is the angle, whoſe ſine is to radius as the ſine of incidence is to the ſine of refraction, out of the denſer medium into the rarer. Thus a ray of light will not paſs out of water into air if the angle of incidence exceeds 48° 36′, the ſine of which is $\frac{3}{4}$ of radius.

A ray of light will not paſs out of glaſs into air if the angle of incidence exceeds 40° 11′, the ſine of which is $\frac{20}{31}$ parts of radius; or out of glaſs into water, if the angle of incidence exceeds about 59° 20′.

EXP. VII.

When a ray of light is refracted by paſſing out of a rarer medium into a denſer, if the angle of incidence be increaſed, the angle of deviation will alſo be increaſed.

Let *i*, *r*, and *d*, repreſent the leaſt cotemporary variations of the angles of incidence, refraction and deviation; we have from the properties of thoſe angles $i = r + d$ and $d = i - r$; but the leaſt increment of incidence is to the cotemporary increment of refraction as the tangent of incidence to the tangent of refraction; hence *i* is greater than *r*; therefore *d* being poſitive, the angle of deviation will of courſe increaſe while the angle of incidence increaſes.

The ſame reaſoning may be applied to demonſtrate, that when a ray of light is refracted by paſſing out of a denſer medium into a rarer, the angle of incidence being increaſed, the angle of deviation will alſo be increaſed.

A priſm is a ſolid terminated by three rectangular parallelograms and two equal and parallel triangles.

A line drawn through the priſm parallel to the interſections of the parallelograms, is called an axis of the priſm.

EXP. VIII.

A ray of light paſſing through a denſer priſm, perpendicular to the axis, is obſerved to be refracted towards the thicker part of the priſm.

The angle contained between the ſides of the priſm through which a ray paſſes, is called the refracting angle.

It is always ſuppoſed, in optical experiments, that when a ray is refracted through a priſm, the incident and emergent rays are perpendicular to the axis.

EXP. IX.

If the angles of incidence in which a ray of light impinges upon the ſurfaces of a glaſs priſm be ſmall, the deviation of the ray from its original courſe, after refraction, will equal about half the refracting angle.

Two angles being diminiſhed, *ſine limite*, may vaniſh in any given ratio, and their ſines will vaniſh in the ſame ratio. This is a mathematical propoſition; but in practice, although two angles be not evaneſcent, yet if they exceed not a few minutes, the ratios of the arcs and of the ſines are ſo nearly the ſame, that as far as regards experiments depending upon them, no ſenſible difference can be obſerved, whether the ratio of the arcs or that of the ſines be aſſumed: thus if the two arcs are 8′ and 4′, the ſines are as 23271 to 11636 nearly; which differs very little from the ratio of 8 : 4.

Cor. I. In general let I = the angle of incidence, R = the angle of refraction, out of the rarer medium into the denſer, and D = the angle of deviation after the two refractions, V = the refracting angle of the priſm; then, if the angles of incidence be very ſmall, $D = \frac{V \times \overline{I-R}}{R}$: in the ſame priſm, therefore, the angle of deviation will remain the ſame, however the angles of incidence upon the ſurfaces (being always ſmall) ſhould vary.

Cor. II. In different priſms formed of the ſame ſubſtance, the angle of deviation D will be proportional to the refracting angles of the priſms.

A double convex lens is a ſolid contained between two ſegments of ſpherical ſurfaces.

A line joining the centers of the ſpherical ſurfaces, is called the axis of the lens.

Exp. X.

There is a point in every double convex lens, through which if a ray paſſes, the incident and emergent rays will be parallel, however the incident ray be inclined to the axis of the lens; this point is called the center of the lens.

If tangents be drawn to the lens, at the points of incidence and emergence in any given plane, theſe tangents will be parallel.

The lens being very thin, or if the direction of the ray's paſſage be not much inclined to the axis, the incident and emergent rays will be nearly in the ſame ſtrait line.

It is to be underſtood, that the lenſes, priſms, &c. uſed in optical experiments, are formed of tranſparent ſubſtances which are denſer than the medium through which the rays flow before they arrive at the refracting ſurfaces. Theſe ſubſtances are uſually glaſs or water.

Exp. XI.

A ray of light falling upon a convex lens is bent by refraction towards the axis, unleſs the inclination of the incident ray to the axis be very great.

A ray of light falling upon any curve ſurface, is refracted or reflected in the ſame manner as if it had fallen upon a plane ſurface which touched the curve in the point of incidence: whence it is the ſame thing, whether a ray is refracted through a lens, or through a priſm, the ſides of which touch the lens in the points through which the ray paſſes.

Exp. XII.

If a ray of light falling nearly parallel to the axis be refracted through a glaſs lens, it will deviate the more from its original direction the more remote from the center, it falls upon the lens.

If the ſides of a priſm be conceived to touch the lens in the points through which the ray paſſes, the angle at the vertex of the priſm will be the greater, the more remote thoſe points are from the axis of the lens; and conſequently, by Exp. IX. the angle of deviation will be the greater.

Exp. XIII.

If a ray of light be refracted through a medium, terminated by parallel ſurfaces, the incident and emergent rays are parallel.

The ſurfaces at the points of incidence and emergence are here ſuppoſed parallel.

A ſlender portion of rays, ſeparated from the reſt, is called a pencil of rays.

Pencils of rays are uſually cylindrical or conical; the axis of a pencil is the ſame as the axis of the cone or cylinder.

Exp. XIV.

Parallel rays falling on a reflecting plane ſurface are reflected parallel.

Exp. XV.

If a pencil of parallel rays falls perpendicularly upon a ſpherical reflector, the focus of reflected rays biſects the diſtance between the ſurface and center.

Foci are thoſe points from which rays of light diverge, or to which they converge.

If parallel rays falling upon a reflecting or refracting ſurface are cauſed to diverge from, or converge to a given point, this point is called the principal focus.

The diſtance between the principal focus and the ſurface of the refractor or reflector, is called the principal focal length.

A ſpherical reflector is ſuppoſed to be a ſegment of a ſpherical ſurface.

A line drawn through the centers of the ſphere, and of the leſſer circle which terminates the ſpherical reflector, is called the axis.

In the determination of focal lengths, the axis of the pencil of incident rays is ſuppoſed to paſs through the centers of the refracting or reflecting ſurfaces, and the mathematical concluſions which follow are only true of thoſe rays which are contiguous to the axes; but experiment admits of a conſiderable deviation from the limitations abovementioned.

Exp. XVI.

If diverging or converging rays fall upon a ſpherical reflecting ſurface, the diſtance of the focus of incident rays from the principal focus, half the radius of the reflector, and the diſtance between the principal focus and the focus of reflected rays, will be in continual proportion.

Suppoſe the reflector to be concave, and the rays to diverge from a focus, the diſtance of which from the ſurface $= d$.

Let the radius of the reflector $= r$; we have by the preceding rule, $d-\frac{r}{2} : \frac{r}{2} :: \frac{r}{2} : \frac{r^2}{4d-2r} =$ the diſtance between the focus of reflected rays and the principal focus.

The focus of reflected rays is in this caſe between the principal focus and the center of the reflector, wherefore adding $\frac{r}{2}$ to the quantity laſt found, we have, $\frac{r^2}{4d-2r}+\frac{r}{2}=\frac{2r^2+4dr-2r^2}{8d-4r}=\frac{dr}{2d-r}$ for the diſtance of the focus of reflected rays from the ſurface.

This ſolution extends to all the caſes of foci formed by reflection from a ſpherical ſurface, by changing the ſign of r when the reflector is convex, and of d when the rays converge to a point, the diſtance of which from the ſurface is d; thus if rays converge upon a concave reflector, the radius of which is 30 inches, and focus of converging rays ſhould be 10 inches from the ſurface, the focal length required will be $\frac{-dr}{-2d-r}=\frac{dr}{2d+r}$ in the preſent caſe $=\frac{30\times 10}{30+20}=6$.

The diſtance between the ſurface of the reflector, and the focus of reflected rays, is called the focal length.

Exp. XVII.

The foci of incident and reflected rays are always on the ſame ſide of the principal focus.

If the focus of incident rays be moved along the axis of the reflector, the focus of reflected rays will move in the oppoſite direction, and the foci will meet at the ſurface and center.

Exp. XVIII.

Parallel rays paſſing through one or more plane refracting ſurfaces, any how inclined, emerge parallel.

Exp. XIX.

The diſtance of the center of a glaſs ſphere, from the principal focus of rays refracted by it, is nearly equal to a radius and a half of the ſphere.

1. In general let $r =$ the radius of the ſphere, the ſines of inſidence and refraction I and R, the diſtance of the principal focus from the center of the ſphere $= \frac{Ir}{2I - 2R}$.

2. Parallel rays paſſing through a globe of water are collected into a focus, the diſtance of which from the center is equal to a diameter of the ſphere.

3. If a pencil of parallel rays falls upon a ſpherical refracting ſurface, we have the following proportion: viz. the difference of the ſines of incidence and refraction is to the ſine of incidence, ſo is the radius of the ſurface to the diſtance between the ſurface and focus of refracted rays; therefore, I and R ſignifying as uſual, and r being the radius of the convex ſurface of a denſer medium, we have the principal focal length $= \frac{Ir}{I - R}$.

Exp. XX.

A pencil of parallel rays falling upon a double convex glaſs lens, terminated by ſurfaces, the radii of which are equal, is obſerved to be collected into a focus, the diſtance of which from the lens is nearly equal to the radius of either ſurface.

If the radii of the ſurfaces be unequal, and the ſines of incidence and refraction I and R, we have the following rule.

Find the principal focal length after refraction from the firſt ſurface, and it will be as the diſtance between the centers of the ſurfaces to the radius of the ſecond ſurface; ſo is the difference between the principal focal length, before found, and the radius of the firſt ſurface, to the focal length required.

Suppoſe the lens to be double convex;
Let the radius of the firſt ſurface $= r$
the radius of the ſecond ſurface $= \varrho$

Then the principal focal length of the firſt ſurface $= \frac{Ir}{I-R}$, and we have from the proportion before mentioned, $r+\varrho : \varrho :: \frac{Ir}{I-R} - r :: \frac{Rr\varrho}{\overline{I-R}\times\overline{r+\varrho}} =$ the principal focal length of the lens.

This ſolution extends to the principal focal lengths of all lenſes, the thickneſs of which is inconſiderable, the ſigns of r and ϱ being changed when the ſurfaces are concave inſtead of convex; if either ſurface is plane, the radius which belongs to it becomes infinite, in which caſe, the finite quantities added to, or ſubſtracted from it, vaniſh.

In glaſs the principal focal length $= \frac{20r\varrho}{11r+11\varrho}$ or $\frac{2r\varrho}{r+\varrho}$ nearly, when $r = \varrho$ as in the experiment, the principal focal length $= \frac{2r^2}{2r} = r$.

Suppoſe the ſecond ſurface of a glaſs lens to become plane, ϱ then is infinite, and $f = \frac{2\varrho r}{r+\varrho} = \frac{2r\varrho}{\varrho} = 2r$, or, the focal length is twice the radius.

The principal focal length is the ſame, whichever ſurface is oppoſed to the incident rays.

Exp. XXI.

Let $d =$ the diſtance between the focus of incident rays and the center of a lens, $r =$ the principal focal length, f the diſtance between the focus of refracted rays and the lens, and we have the following proportion: as $d-r : r$, ſo is $d : f$.

If the two radii of a glaſs lens be r and ϱ, it appeared from the laſt note that the principal focal length will be $\frac{20r\varrho}{11r+11\varrho}$, or $\frac{2r\varrho}{r+\varrho}$, ſufficiently accurate for moſt experiments. Let $d =$ the diſtance of a focus of incident rays, f the diſtance of the focus of emergent rays from the lens, and we have the following proportion, as $d - \frac{2r\varrho}{r+\varrho} : \frac{2r\varrho}{r+\varrho} :: d : f$ wherefore $f = \frac{2dr\varrho}{dr+d\varrho-2r\varrho}$.

Every thing elſe remaining, the focal length will be the ſame; whichever ſurface of the lens be oppoſed to the incident rays.

If the lens ſhould be of any ſubſtance in which the ſines of incidence and refraction are as $I : R$, inſtead of $\frac{2r\rho}{r+\rho}$, we muſt ſubſtitute the general expreſſion $\frac{Rr\rho}{\overline{I-R}\times\overline{r+\rho}}$, and the proportion will become $d - \frac{Rr\rho}{\overline{I-R}\times\overline{r+\rho}} : \frac{Rr\rho}{\overline{I-R}\times\overline{r+\rho}} : d : f$, whence $f = \frac{Rdr\rho}{\overline{I-R}\times\overline{dr+d\rho}-Rr\rho}$, which is a general ſolution for all caſes of focal lengths when the thickneſs of the lens is inconſiderable.

Having determined the properties relating to the foci of ſingle pencils of rays, we proceed to conſider in what manner the combination of the foci of ſeparate pencils forms images or pictures of the objects from which the rays flow.

EXP. XXII.

Images formed by reflexion from a plane ſurface, are erect, ſimilar, and equal to the objects, and are as far diſtant behind the ſurface as the objects are before it.

EXP. XXIII.

An object being immerſed in water, appears nearer to the eye than it really is by $\frac{1}{4}$ of its depth from the water's ſurface.

In general if rays diverge from an object, and fall upon a refracting ſurface, out of a denſer medium into a rarer, the diſtance of image will be to the diſtance of the object, from the refracting ſurface, as the ſine of incidence to the ſine of refraction, out of the denſer medium into the rarer.

EXP. XXIV.

If an object be the arc of a circle, having the ſame center with a ſpherical reflector, the image will be a ſimilar concentric arc; and the magnitudes of the image and object will be to each other in the ſame proportion as their diſtances from the center of the reflector.

Having given $r =$ radius of a concave reflector, $d =$ the diſtance of the object from the ſurface, we have the diſtance of the image $= \frac{dr}{2d-r}$, and the magnitudes of the object and the image, as $2d-r : r$.

The diſtances of the object and image, from the center of the reflector, are in the ſame proportion as their diſtances from the ſurface.

Exp. XXV.

Every thing remaining as in the laſt experiment, the image is obſerved to be inverted, or erect, with reſpect to the object, according as they are on different, or the ſame ſide of the center of the reflector.

If the object is moved along the axis of the reflector, the image will be moved in a contrary direction.

Exp. XXVI.

An object ſeen by reflexion from a ſpherical convex ſurface, appears diminiſhed.

The apparent magnitude depends on the angle which the image ſubtends at the eye; in the preſent caſe the image being nearer to the center than the object, will be leſs than the object in the ſame proportion.

Exp. XXVII.

Images of plane objects, when ſeen by reflexion from a cylindrical poliſhed ſurface are not ſimilar to the objects; likewiſe plane objects apparently diſtorted appear, from the aforementioned reflexion, in their proper proportion.

Exp. XXVIII.

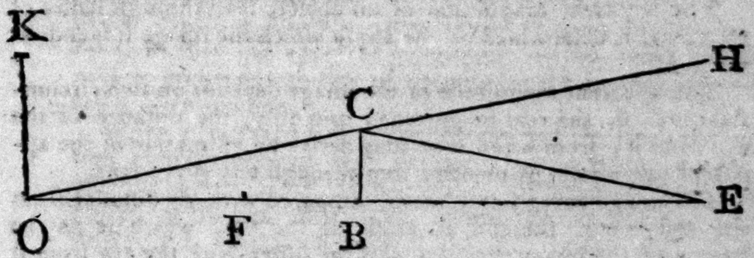
If the arc of a circle, the center of which coincides with the center of a double convex lens, be conſidered as an object, the image will be an arc ſimilar to the object, and the magnitudes of the object and image will be to each other, as their diſtances from the center of the lens.

The properties (with regard to focal length) of all lenſes, the principal focal length of which is given, are the ſame. Wherefore, let r be the principal focal length, d the diſtance of an object from the lens; then, whatever be the radii of the ſurfaces, the diſtance of the image from the lens $= \frac{dr}{d-r}$.

Every thing elſe remaining, no difference is obſerved in the focal length, whichever ſurface is oppoſed to the incident rays.

The magnitude, poſition, and diſtance of the image from the lens remain the ſame, whatever be the figure of the aperture through which the rays flow.—The brightneſs of the image will be as the area of the aperture when the magnitude of the image is given.

The image of a right line formed by a lens is not rectilinear, but coincides with one of the conic ſections, which is determined by the following conſtruction.



Let *B* repreſent the center, *OE* the axis and *FB* the principal focal length of a double convex lens, let *OK* be a right line drawn perpendicular to the axis, be conſidered as an object.

Draw $BC=BF$, at right angles to *BO*: Join *CO* and produce it, at the point *C* deſcribe the angle $HCE=BCO$. *B* and *E* will be the foci of the conic ſection which coincides with the image, the major axis of which $=\overline{BC+CE}$.

Cor. I. The point *F* being between *B* and *O*, the ſection is an ellipſe.

Cor. II. If *O* coincides with *F*, *OCB* and *ECH* will be each 45 degrees, and *BCE* a right angle, wherefore *CE* will be parallel to *BE*, or *E* will be at an infinite diſtance. The conic ſection is in this caſe a parabola.

Cor. III. If *OB* is leſs than *BC*, *BCE* will become greater than a right angle, and conſequently *E* will now be removed to the oppoſite ſide of *B*, in which caſe the ſection will be an hyperbola; theſe properties follow from the principles of conic ſections and of focal lengths.

Cor. IV. Suppoſe $OB=d$, $BC=p$, we have likewiſe from the properties of conics, the ſemiaxis major of the ſection $=\frac{d^2p}{d^2-p^2}$, and the ſemiaxis minor $\frac{dp}{\sqrt{d^2-p^2}}$.

Cor. V. The principal parameter of the ſection is always equal to twice the principal focal length of the lens.

Cor. VI. The image of a very diſtant right line is a circle, the radius of which is equal to the principal focal length of the lens.

Cor. VII. Whatever be the magnitude or figure of the image of a right line formed by a given lens, that part of it which lies contiguous to the axis will always have the ſame curvature as a circle, the radius of which is equal to the principal focal length of the lens.

Theſe principles are eaſily applied to images of a right line formed by reflexion from ſpherical ſurfaces.

EXP. XXIX.

Objects ſeen through a double concave lens appear diminiſhed.

The apparent magnitude of an object, ſeen through lenſes of all kinds, is determined by the angle which the image ſubtends at the eye.

The apparent magnitude of the image depends on two circumſtances: ſt. the real magnitude; and 2dly, the diſtance of the eye from it; from whence we may derive an eſtimation of the apparent magnitude of objects, ſeen through any given lens.

The image being ſmall when compared with its diſtance from the eye, will ſubtend an angle at the eye, which is as the length of the image directly, and the diſtance of the eye from it inverſely.

Let $d=$ the diſtance of an object from a double concave lens, $r=$ the principal focal length; the diſtance of the image from the lens will be $\frac{dr}{d+r}$ on the ſame ſide with the object; let $e=$ the diſtance of the eye from the lens; then, according to the preceding principles, the angle ſubtended by the object, when ſeen with the naked eye, will be to that which it appears under, when viewed through the lens, as $\frac{d}{d+e}$ to $\frac{dr}{d+r}$ divided by $e+\frac{dr}{d+r}$, or as $dr+er+ed:dr+er$.

It is evident by inſpection that the latter quantity muſt be ſmaller than the former; for which reaſon the object will appear diminiſhed, unleſs the lens be contiguous to the object or to the eye.

Exp. XXX.

An object ſeen through a double convex lens will appear magnified, if its diſtance from the lens be leſs than the principal focal length.

Let $d =$ the diſtance of the object from the lens; e the diſtance of the eye; r the principal focal length: then the angle ſubtended by the object when ſeen by the naked eye, will be to that under which it appears, when viewed through the lens, as $re+dr-ed$ to $re+dr$, the latter of which quantities, being always greater than the former, the object muſt appear magnified, unleſs the lens be contiguous to the object or to the eye.

Thus if the principal focal length of a reading glaſs be 12 inches, the diſtance of the lens from an object ſeen through it 4 inches, and from the eye 6; the object will appear magnified in the proportion of $12\times6+4\times12$ to $12\times6+4\times12-4\times6$, or of 5 to 4.

The ſame proportion obtains when the object being placed at a greater diſtance from the lens than the principal focal length, the eye is ſituated between the lens and the image wherefore, when $d=r$, that is, when the object is placed in the principal focus, the angle ſubtended by it at the naked eye, will be to that under which it appears when viewed through the lens, as r to $r+e$, ſo that the apparent magnitude of the image will bear a greater proportion to that of the object, the further the eye recedes from the lens.

When the image is between the eye and the lens, the proportion abovementioned will become $de-er-dr$ to $er+dr$, and when d is very great, as $e-r$ to r. Wherefore, a very diſtant object, as the ſun, moon, &c. will appear neither magnified nor diminiſhed, when ſeen through a double convex lens, if the diſtance of the eye from the lens, be equal to twice the principal focal length; if the diſtance of the eye be greater the object will appear diminiſhed, if leſs, magnified.

The image and object will be erect or inverted, with regard to each other, according as they are on the ſame or different ſides of the lens: hence all real images muſt be inverted with reſpect to the objects.

If r repreſents the principal focal length of the lens, d its diſtance from the object, then the lineal magnitude of the image will be to that of the object as $\frac{dr}{d-r} : d$, or as $r : d-r$.

Cor. It follows from hence, that if the object be placed at a diſtance of twice the principal focal length from the lens, the image will be formed at the ſame diſtance, and will be therefore equal to the object.

Exp. XXXI.

The angle under which a very diſtant object is ſeen, through a teleſcope conſiſting of two double convex lenſes, is to the angle, under which the ſame object is ſeen by the naked eye, as the principal focal length of the object glaſs is to the principal focal length of the eye glaſs.

In this ſort of teleſcope, the diſtance between the two lenſes is equal to the ſum of their principal focal lengths.

Let the diſtance of the eye from the object glaſs be e, the principal focal length of the object glaſs $= r$; then, by the note to Exp. XXX, the angle ſubtended by the object, when ſeen by the naked eye, will be to that under which it appears through the teleſcope, as $e-r : r$; that is, as the principal focal length of the eye glaſs to that of the object glaſs.

Exp. XXXII.

A minute object, when ſeen through a lens of very ſmall principal focal length, appears magnified and diſtinct, if the object be placed in the principal focus.

The angle under which the object appears, will be to that which it ſubtends when ſeen with the naked eye, as the diſtance at which it is viewed by the naked eye diſtinctly, to the principal focal length of the lens.

If the principal focal length of the lens does not exceed $\frac{1}{50}$ of an inch; and the diſtance at which the eye can ſee diſtinctly be about ſix inches; it follows that the lens will magnify about 300 times in diameter.

In this experiment, the rays flowing from any point in the object will emerge from the lens parallel, and being collected into a focus upon the retina of the eye, will render the image diſtinct: the uſual ſtructure of the eye is ſuch as to collect rays, which fall upon it nearly parallel, into a focus upon the retina, whereon the images of objects from which the rays flow are diſtinctly formed.—It ſometimes happens that the eye, by too great a degree of convexity, cauſes each pencil to converge to a focus before it arrives at the retina, and conſequently, the rays which flow from each point of an object, are not collected into a correſponding point upon the retina, but are diſperſed over a ſmall circle, whereby the object is imperſectly repreſented upon the retina, and the ſmall circles above deſcribed, being intermixed, increaſe the confuſion of viſion.—The application of a concave lens, properly adjuſted to the eye, removes the focus of each pencil further from the object, ſo as to fall exactly upon the retina.

A contrary defect ariſes when the eye becomes ſo flattened as to collect the rays into foci behind the retina, cauſing a confuſion of viſion ſimilar to the former. This is remedied by a convex lens, which aſſiſts the eye in collecting the rays, bringing the foci of the ſeveral pencils nearer to the object, and thus, by forming the image upon the retina, renders viſion diſtinct.

As to the ſtructure of the eye (the humours of which conſtitute a compound lens) and the uſes of its various parts; anatomical diſſection will convey more adequate ideas than can be derived from deſcription.

EXP. XXXIII.

Small objects may be ſeen diſtinct and magnified through a combination of two or more lenſes.

Theſe combinations of lenſes are called compound microſcopes, of which there is a great variety of conſtructions.

EXP. XXXIV.

If a minute tranſparent object, illumined by the rays of the ſun, be placed before a ſmall lens, at a little greater diſtance from it than the principal focus, the image will be deſcribed on a ſcreen, properly placed, diſtinct and magnified.

If the diſtance of the object from the lens be d, and the principal focal length of the lens $= r$, the diſtance of the ſcreen or image from the lens will be $\frac{dr}{d-r}$, and the lineal magnitude of the object will be to that of the image as $d : \frac{dr}{d-r}$, or as $d-r : r$.

Upon this principle the ſolar microſcope is conſtructed.

EXP. XXXV.

A ray of the ſun's light being refracted through a glaſs priſm, is obſerved to be ſeparated into rays of different colours.

Theſe colours are red, orange, yellow, green, blue, indigo and violet, and are called the priſmatic colours from the experiment. The ſeparated rays deviate unequally from the courſe of the incident ray, whence they are ſaid to be differently refrangible. The refrangibility of the rays anſwers to the order in which the colours appear in the priſmatic ſpectrum, the red being the leaſt, and the violet ray the moſt refracted: either of the ſeven colours ſeparated from the reſt is called an homogeneal ray, or homogeneal light.

Exp. XXXVI.

Homogeneal light cannot be further ſeparated or altered, as to its colour, by refraction through any number of priſms.

Exp. XXXVII.

A ray of light, paſſing through an uniform medium, terminated by parallel ſurfaces, is not at all ſeparated into colours.

It is always obſerved, that when the homogeneal rays, after having been ſeparated from a common ray of light, are cauſed by ſubſequent refraction to emerge parallel, the emergent rays are colourleſs; in this experiment, the homogeneal rays emerge after the ſecond refraction parallel.

Exp. XXXVIII.

Thoſe rays which are the moſt refrangible are alſo the moſt reflexible.

The ſine of incidence is to the ſine of refraction in the red rays, paſſing out of glaſs into air, as 50 : 77; wherefore, there will be reflected from the ſurface of glaſs, when the angle of incidence exceeds 40° 29′, the ſine of which is $\frac{50}{77}$ of radius; the ſine of incidence is to the ſine of refraction of the violet rays, as 50 : 78; thoſe rays, therefore, will be reflected if the angle of incidence exceeds 39° 52′, the ſine of which is $\frac{50}{78}$ of radius.

Exp. XXXIX.

The homogeneal rays, being collected into a focus upon a ſcreen of white paper, are colourleſs.

Exp. XL.

If one of the homogeneal rays be ſtopped before it reaches the ſcreen, the ſpectrum conſiſting of the remaining rays becomes coloured.

It appears from the preceding Experiment, that ſolar light is an heterogeneal ſubſtance, conſiſting of ſeven ſorts of homogeneal rays.—Let a ray of ſolar light be refracted out of air into any given medium; the homogeneal rays will impinge upon the refracting ſurface at a common angle of incidence, but the ſines of the angles in which the rays are refracted will be different; the angle contained between the red and violet rays, after refraction, is called the angle of diſſipation, and is the difference of the laſt angles in which thoſe rays are refracted.

Let the ſine of incidence be to the ſine of refraction of the rays of mean refrangibility, paſſing out of air into any medium, as $m : 1$, of the violet rays as $m + \dot{m} : 1$, and of the red as $m - \dot{m} : 1$.

The quantity $\dot{m}$ being conſtant in the ſame medium, is therefore aſſumed as a meaſure of the diſperſing or diſſipating power. Thus the ſine of incidence is to the ſine of refraction in the rays of mean refrangibility, paſſing out of air into common glaſs, as $\frac{77.5}{50} : 1$, of the red rays as $\frac{77}{50} : 1$, and in the violet as $\frac{78}{50} : 1$, wherefore, putting $\frac{77.5}{50} = m$, we have $\frac{78}{50} = m + \dot{m}$ and $\dot{m} = \frac{1}{100}$. In the ſame manner, if the ſine of incidence be to the ſine of refraction of the rays of mean refrangibility, paſſing out of air into water, as $\frac{108.5}{81} : 1$, of the red rays as $\frac{108}{81} : 1$, and of the violet rays as $\frac{109}{81} : 1$, and w be made $= \frac{108.5}{81}$, we ſhall have $\dot{w} = \frac{1}{162}$, and $\dot{m} : \dot{w}$, or the ratio of the diſperſing powers exiſting in the two mediums, whereby they ſeparate the homogeneal rays, is as $\frac{1}{100} : \frac{1}{162}$, or as $81 : 50$.

Suppoſe that the ſine of incidence is to the ſine of refraction of red rays, paſſing out of air into flint glaſs, as $1.565 : 1$, of the violet as $1.595 : 1$; then the ratio of refraction in the rays of mean refrangibility is as $1.58 : 1$, and if $n = 1.58$, $\dot{n} = \frac{3}{200}$, and the ratio of the diſperſing powers of flint and common glaſs will be as $\frac{3}{200} : \frac{1}{100}$, or as $3 : 2$.

Exp. XLI.

A ray of ſolar light being refracted thro' the ſides of an iſoſceles priſm of common glaſs, the refracting angle of which is 30°,

when the rays of mean refrangibility paſs parallel to the baſe, the angle of diſſipation will be about 39′.

If a ray of ſolar light be refracted through one ſurface only, let $t =$ the tangent of refraction of the rays of mean refrangibility, $m : 1$ the ratio of refraction of the ſame rays, $\dot{m}$ the meaſure of the diſſipating power; the angle of diſſipation will be ſubtended by an arc $= \frac{2\dot{m}t}{m}$, radius being 1.

Thus if a ſolar ray impinges upon a ſurface of common glaſs, at an angle of incidence $= 20^\circ$, m being $= \frac{77.5}{50}$, or 1.55, the angle of refraction of the rays of mean refrangibility $= 12^\circ.44' 52''$, and the arc which meaſures the angle of diſſipation $= \frac{2 \times \text{tang. } 12^\circ.44'.52''}{100 \times 1.55} = 10'.3''$, radius being $= 1$.

If the rays are refracted through two ſurfaces inclined to each other, as the ſides of a priſm, let the ſine of the refracting angle $= a$, the coſine of the angle of refraction of the rays of mean refrangibility at the firſt ſurface $= p$, the coſine of refraction at the ſecond ſurface $= q$, then the angle of diſſipation, at which the violet and red rays are inclined to each other, after the ſecond refraction, $= \frac{2\dot{m}a}{pq}$. Thus in the experiment $\dot{m} = \frac{1}{100}$; $a =$ the ſine of 30°, $p =$ coſine of 15°, $q =$ coſine $23^\circ.39'.5''$, and the arc which meaſures the angle of diſperſion (rad. being 1) $= \frac{2 \times \text{ſine } 30^\circ}{100 \times \text{coſ. } 15^\circ \times \text{coſ. } 23^\circ.39'.5''} = 38'.51''$.

If a ray be refracted through parallel ſurfaces, the angle of diſſipation vaniſhes, becauſe a, or the ſine of the refracting angle, is in that caſe nothing.

Exp. XLII.

Let the vertex of a flint glaſs priſm, the refracting angle of which $= 23^\circ.40'$, be applied to the baſe of a common glaſs priſm, the refracting angle of which $= 25^\circ$; a ray of ſolar light will paſs directly through the priſms, when their ſurfaces are contiguous, but the emergent ray will be coloured.

It was formerly imagined, that a ray of ſolar light would, in all caſes, emerge colourleſs after refraction, if it did not deviate from the courſe of the incident ray, the contrary to which, however, appears from this experiment.

The ray is ſuppoſed to fall perpendicularly upon the ſurface of the priſm, the refracting angle of which is the greateſt.

The poſition of the priſms in the Experiment is ſuch, that the effects of refraction upon the paralleliſm of the homogeneal rays paſſing through them are contrary to each other, and conſequently if they were equal the rays would emerge parallel: but the flint priſm by its greater diſſipating power more than counteracts the ſeparation of the rays cauſed by their paſſage through the firſt priſm, which = $38'\frac{1}{2}$, and, inverting the order of the colours, cauſes the red and violet rays to emerge inclined to each other, at an angle of $12'\frac{1}{4}$, which is ſufficiently great to produce a ſenſible tinge of the priſmatic colours in the emergent rays.

The difficulty which chiefly impeded the improvement of teleſcopes was, to refract a ray ſo, that while it ſhould deviate conſiderably from its original courſe, the diſperſion of the homogeneal rays might be counteracted, and that by this means they might all emerge parallel, and of courſe free from colour, which is not by any means to be effected, except by the combination of tranſparent ſubſtances, the refracting and diſſipating powers of which are different.

EXP. XLIII.

Every thing remaining as in the laſt Experiment, let the vertex of a common glaſs priſm, the refracting angle of which is 10°, be applied to the baſe of the flint priſm; if a ray of ſolar light paſſes through the three priſms, when their ſurfaces are contiguous, the emergent ray will deviate about 5°. 37′ from the courſe of the incident ray, but will be colourleſs.

In this caſe the two common glaſs priſms refracting the ray in the ſame direction, cauſe it to deviate from the courſe of the incident ray about 5°.37′ more than the deviation in the contrary direction, ariſing from refraction through the flint priſm; but the latter, by its greater diſſipating power, exactly counteracts the ſeparation of the rays occaſioned by refraction through the other two priſms, ſo that the homogeneal rays emerge at length parallel, and of courſe colourleſs.

In this Experiment the rays of mean refrangibility emerge at an angle of refraction = 16°.57′. If a ſolar ray impinged upon the ſurface of the priſm laſt applied, at an angle of incidence = 16°.57′, the angle of diſſipation, after emergence into air, would be $12'\frac{1}{4}$.—It appeared from the laſt Experiment, that the diſſipation of the rays emerging from the two priſms = $12'\frac{1}{4}$;

for which reaſon (and on account of the contrary poſition of the priſms) the red and violet rays emerging inclined to each other at an angle of 12′ ¼ from the two priſms, and falling upon the ſurface of the third, will be refracted out of it colourleſs.

Exp. XLIV.

If a ray of ſolar light paſſes directly through any two priſms, the diſſipating powers of which are unequal, it will be ſeparated into the homogeneal rays.

Although the deviations of the ray, after refraction at each priſm, be equal and contrary, yet one of the priſms poſſeſſing a greater power of diſſipating the ray than the other, the homogeneal rays emerge inclined to each other, and conſequently the emergent ray becomes coloured.

The aperture of a lens is terminated by the circumference of a circle, the center of which coincides with the axis of the lens; the circumference abovementioned is always ſuppoſed to be contiguous to the lens, and conſequently its plane is perpendicular to the axis.—The annulus of the lens, which is more remote from the axis than the circumference which terminates the aperture, is ſuppoſed to be covered with an opake ſubſtance. If a point, or in practice, any ſmall luminous [illegible], ſituated in the axis of the lens, be conſidered as an object, the image which is formed by the rays flowing from it through the leaſt aperture, ſufficient for viſion, is called the principal image.

Body

If all the aperture be covered with an opake ſubſtance, except an annulus adjacent to its circumference, the image formed by rays paſſing through this annulus, is called the extreme image.

Exp. XLV.

If homogeneal rays flow from an object placed in the axis of a double convex glaſs lens, the extreme image of the object will be nearer to the lens than the principal image.

Hence it appears, that the rays which diverge from a given point in an object, are not all collected into a point in the correſponding image, many images of the point being diffuſed over the ſpace contained between the principal and extreme images.—The diſtance between the principal and extreme images, is called the ſpace of diffuſion.

This aberration of the rays from the geometrical focus, ariſing from the ſpherical figure of the refracting ſurfaces, is one of the cauſes of indiſtinct viſion in teleſcopes, &c.

Exp. XLVI.

The extreme image of an object, formed by a double convex lens, is coloured and indiſtinct, if the proportion of diameter of the aperture to the principal focal length exceeds a certain limit.

The colour and indiſtinctneſs which ~~is~~ are here obſerved in the extreme image, ariſe from the different refrangibility of the homogeneal rays compoſing ſolar light, which emerging from the lens with too great a degree of obliquity, becomes ſenſibly ſeparated into the coloured rays; theſe form many images of the ſame point, all of which are at different diſtances from the lens.—This aberration cauſes an indiſtinctneſs even in the principal image, but in all the others, formed by rays flowing through the lens more remote from the axis, the indiſtinctneſs is very much increaſed.

The two ſorts of aberration above deſcribed are the chief impediments to the perfection of teleſcopes.

The method uſed for obviating theſe difficulties will appear in the following experiments.

Exp. XLVII.

An object glaſs may be compounded of three lenſes, whereof two are double convex, made of common glaſs, incloſing a double concave of flint glaſs, ſo that the extreme and principal images of objects formed by it ſhall nearly coincide.

The concave glaſs being of greater refracting power than the other two, will diminiſh the angle of deviation of the extreme rays, more than of thoſe which are nearer to the axis; the refracted extreme rays will, therefore, interſect the axis further from the center of the compound lens than if it had paſſed through a double convex lens only of the ſame principal focal length.

The radii of the lenſes abovementioned may be ſo adjuſted that the ſpace of diffuſion ſhall vaniſh.

Exp. XLVIII.

The refracting and diſſipating powers of the three lenſes, which conſtitute a compound object glaſs, being given, the radii of the ſurfaces may be ſo adjuſted to each other, that the extreme, principal, and the intermediate images of objects formed by it ſhall be diſtinct and colourleſs.

It appears from the preceding Experiments, that the ſame combination of lenſes, not only reduces the ſpace of diffuſion into an inſenſible quantity, but at the ſame time refracts the extreme rays to the geometrical focus without ſeparating them into the primary colours, whereby the aberration ariſing from the different refrangibility of the rays, as well as from the ſpherical figure of the lens, is almoſt wholly removed.—Lenſes, conſtructed upon theſe principles, have been called perfect.

A difficulty has ariſen in the conſtruction of perfect lenſes, viz. to determine the power of diſſipating the homogeneal rays in different refracting ſubſtances, which does not obſerve any law of the refracting power, denſity, or of any quantities formed out of them.

Exp. XLIX.

Let a common glaſs priſm, the refracting angle of which = 30°, be applied contiguous to a priſm of flint glaſs, the refracting angle of which = 19°; let the vertices of the priſms be placed in oppoſite directions, a ſolar ray being refracted thro' them will deviate from the courſe of the incident ray, but will not be ſeparated into the coloured rays.

In this Experiment the angle of incidence upon the firſt ſurface of the common glaſs priſm being $= 0$, the angle of refraction at which the ray emerges from the flint priſm $= 16^\circ.31'\tfrac{3}{4}$, and the angle contained between the incident and emergent rays $= 5^\circ.31'\tfrac{3}{4}$.

If a ſolar ray is refracted through the common glaſs priſm only, the angle of incidence being $= 0$, the angle of refraction at which the middle rays emerge $= 50^\circ.48'.18''$, wherefore, the angle of diſſipation will be meaſured by an arc (rad. being 1) = $\frac{2 \times \text{ſine } 30^\circ}{100 \times \text{coſ. } 50^\circ.48'.18''} = 54'.24''$, by the rule in note to Experiment XLI.

In like manner if a ſolar ray impinges upon the flint priſm at an angle of incidence $= 16^\circ.31'\tfrac{3}{4}$, it will emerge at an angle of refraction $= 50°.48'.18''$, and it is inferred from hence that the angle of diſſipation after emergence will be meaſured by an arc (radius being 1) $= \frac{3 \times \text{ſine } 19°}{100 \times \text{coſ. } 50°.48'.18'' \times \text{coſ. } 10°.22'.26''} = 54'$.

The angles of diſſipation above determined differ only 24″, which being imperceptible in experiments, the angles are, phyſically ſpeaking, equal.

Thus it appears that the two priſms operate equally upon the paralleliſm of the homogeneal rays, paſſing through them in the directions above deſcribed, and theſe effects by the poſition of the priſms tending to correct each other, the homogeneal rays will emerge, after being refracted through them, parallel and colourleſs.

As to determining the powers of diſſipation and refraction exiſting in different refracting ſubſtances, the following method ſeems as eligible as any.

Suppoſe a ſolar ray to paſs through a priſm (the refracting and diſſipating powers of which are required) impinging perpendicularly upon the firſt ſurface.—Let the angle of diſſipation and the angle of refraction of the middle rays be accurately meaſured, and let $a =$ the meaſure of the angle of diſſipation, $t =$ the tangent of refraction (radius being 1.) The angle of incidence upon the ſecond ſurface being the complement of the refracting angle of the priſm to 90°, and the angle of refraction being determined as above, we obtain the ratio of refraction: if this be as $n : 1$, then the meaſure of the diſſipating power $\dot{n} = \frac{an}{2t}$.

It is to be remarked that the preceding rules concerning the angle of diſſipation are mathematically true, only when $\frac{\dot{n}}{n}$ is leſs than any aſſignable quantity; but in the practical application of them to ſuch refracting ſubſtances as are commonly uſed in optics (eſpecially when the angles of refraction are ſmall) they will not differ from the truth more than a few ſeconds of a degree, the effects of which minute error are inſenſible in experiment.

Exp. L.

Coloured ſubſtances reflect the homogeneal rays of light, which are of the ſame colour with themſelves, more copiouſly than the other rays.

A red ſubſtance held in green rays, if all other light be perfectly excluded, appears green, though very faintly; when held in the red rays, it appears of a very vivid red, which ſhews that the ſubſtance is of ſuch a nature, as to reflect the red rays much more copiouſly than the green.

Exp. LI.

Let a circle be divided into ſeven ſectors, proportional to the ſpaces occupied by the colours in the priſmatic ſpectrum, and let theſe ſectors be painted with the primary colours, according to their proper order and proportion; if the circle be turned ſwiftly in its own plane round the center the whole will appear nearly white.

The artificial colours being leſs pure than the natural, the mixture of the ſeven colours in the experiment will not produce a perfect whiteneſs, but will approach nearer to it, the more care is taken to divide the circle in the true proportions, and the more nearly the colours made uſe of approach thoſe which compoſe a ray of light.

EXP. LII.

The heat produced by collecting the rays of the ſun into the principal focus of a concave mirror or double convex lens, is obſerved to be the more intenſe the greater the area of the mirror or lens, and the leſs that of the focus into which the rays are collected.

The intenſity of the heat produced by burning mirrors or lenſes, is proportional to the ſquares of their diameters directly, and the ſquares of their principal focal lengths inverſely.

Hence the intenſity of the ſun's rays, collected into the principal focus of a concave mirror, will be the ſame as in the principal focus of a double convex lens of the ſame area, the principal focal length of which is one half the radius of the reflector.

EXP. LIII.

Rays of the ſun, however condenſed, communicate no heat to an uniform medium through which they paſs.

EXP. LIV.

If a ray of light falls upon a glaſs ſphere, and emerges after one reflexion and two refractions; the priſmatic colours will be moſt viſible in thoſe emergent rays, which are inclined to the incident rays at an angle of 17° 50′.

The violet rays will appear moſt copiouſly at the angle mentioned in the Experiment, but the red rays will appear when the incident and emergent rays are inclined at an angle of about 19°.25′: from whence it follows, that the breadth of a rainbow, formed by one reflexion and two refractions through ſpheres of glaſs, would appear by obſervation = 1°.35′+32′=2° 7′.

It may be uſeful to ſubjoin a few notes concerning the rainbow.

1. The firſt rainbow is that which is moſt uſually ſeen in the heavens, and is formed by one reflexion and two refractions of the ſun's rays falling upon drops of rain; the ſecondary rainbow is formed by the rays falling upon drops of rain, and emerging after two refractions and two reflexions.—Theſe bows are never ſeen but when it rains and the ſun ſhines at the ſame time, the drops of rain and the ſun being, with regard to the ſpectator's horizon, in oppoſite parts of the heavens.

2. The apparent ſemi diameter of the bow is equal to the apparent altitude of the higheſt point of the bow, together with the altitude of the ſun's center above the horizon.

3. The apparent ſemi-diameter of the firſt rainbow is the difference between four times the angle of refraction and twice the angle of incidence of thoſe contiguous rays which fall upon the drops of rain and emerge parallel after two refractions and one reflexion; this angle, belonging to the red rays, is about 42°. 2′; hence no primary rainbow can be obſerved unleſs the altitude of the ſun's center above the horizon is leſs than 42°. 2′.—For the ſake of brevity, the rays are here conſidered as flowing from the ſun's center only.

4. Contiguous rays, falling upon a refracting globe, emerge parallel after two refractions and one reflexion, when the leaſt increment of the angle of incidence is to the cotemporary increment of the angle of refraction as 2 : 1.

5. Henee the tangent of incidence is to the tangent of refraction as 2 : 1.

6. To find thoſe two angles, the ſines of which are in the proportion of 4 : 3, and tangents as 2 : 1.

Let x and y = the coſines of incidence and refraction reſpectively, then the ſquares of their ſines are $1-x^2$ and $1-y^2$, radius being unity, and the ſquares of their tangents will be $\frac{1-x^2}{x^2}$ and $\frac{1-y^2}{y^2}$, whence we have, by the conditions of the problem

$$1-x^2 : 1-y^2 :: 16 : 9$$

and $\frac{1-y^2}{y^2} : \frac{1-x^2}{x^2} :: 1 : 4$, and joining theſe ratios

$$\overline{x^2 : y^2 :: 4 : 9}$$

and $x^2 = \frac{4y^2}{9}$, wherefore

$1-x^2 = \frac{9-4y^2}{9}$; whence, referring to the firſt proportion,

$\frac{9-4y^2}{9} : 1-y^2 :: 16 : 9$, and multiplying extremes and means

$9-4y^2=16-16y^2$, whence $y^2=\frac{7}{12}$ and $y=\sqrt{\frac{7}{12}}=$ coſine of the angle of refraction; hence we determinep the angle of refraction $=40^\circ.12'.11''$, and the angle of incidence (being that the ſine of which is to the ſine of $40^\circ.12'.11''$, as 4 : 3) $=59^\circ.23'.28''$ which gives us $\frac{1}{2}$ the angle contained between the incident and emergent rays, or $\frac{1}{2}$ the ſemi-diameter of the bow $=2\times\overline{40^\circ.12'.11''}-59^\circ.23'.28''=21^\circ.0'.54''$; wherefore, the angle between the incident and emergent rays, or the apparent ſemi-diameter of the bow, formed by the red rays, $=42^\circ.1'.48''$, or about $42^\circ.2'$.

In the ſame manner the apparent ſemi-diameter of the rainbow, formed by the violet rays, is found to be about $40^\circ.17'$, which being ſubtracted from $42^\circ.2'$, leaves $1^\circ.45'$ for the apparent breadth of the bow. This would be the breadth if the ſolar rays flowed from the center of the ſun only; and ſince the apparent diameter of the ſun is about $32'$, the apparent breadth of the bow will be increaſed by $32'$, ſo that it will appear by obſervation $=2^\circ.17'$.

The following rules will extend to determines the angles of incidence and refraction of contiguous homogeneal rays, emerging parallel from refracting ſpheres of any kind, the number of reflexions within the ſpheres being in general $=n$.

1. The contiguous rays will emerge parallel after n reflexions and two refractions, when the leaſt variation of the angle of incidence is to the cotemporary variation of the angle of refraction as $n+1:1$.

2. Suppoſe $n+1:1$ as $1:t$, hence the tangent of incidence muſt be to the tangent of refraction as $1:t$; let the ſine of incidence be to the ſine of refraction as $1:s$, then to find the angles of incidence and refraction, call their coſines x and y, and we have from the conditions of the problem,

$1-x^2:1-y^2::1:s^2$ radius being unity,

and $\frac{1-y^2}{y^2}:\frac{1-x^2}{x^2}::t^2:1$, joining theſe ratios,

$$x^2:y^2::t^2:s^2$$

and $x^2=\frac{t^2y^2}{s^2}$, and $1-x^2=\frac{s^2-t^2y^2}{s^2}$, ſubſtituting this value of $1-x^2$ in the firſt proportion, we have $\frac{s^2-t^2y^2}{s^2}:1-y^2::1:s^2$,

Whence $1-s^2=y^2-y^2t^2$

and $y^2=\frac{1-s^2}{1-t^2}$, and $y=\sqrt{\frac{1-s^2}{1-t^2}}=$ coſine of refraction in general. The ſine of refraction $=\sqrt{\frac{s^2-t^2}{1-t^2}}$, and conſequently the ſine of incidence $=\sqrt{\frac{s^2-t^2}{1-t^2}}\times\frac{1}{s}$.

The angle of incidence being determined $=I$, and the angle of refraction $=R$; let $p=90^\circ$, $m=$ the number of reflexions $+1$; then half the angle contained between the incident rays and thoſe which emerge contiguous and parallel will be $=$ $\overline{3-m}\times p+mR-I$.

ASTRONOMY.

1. ASTRONOMY is a branch of natural knowledge, which relates to appearances in the heavens and the cauſes of them.

2. Plain Aſtronomy is that part of this ſcience which deſcribes the phenomena and their immediate cauſes only; Phyſical Aſtronomy demonſtrates the firſt known cauſes of the phenomena.

3. A ſpectator viewing the heavens, conceives himſelf to be ſituated in the center of a ſphere, upon the concave ſurface of which all the heavenly bodies are diſpoſed.

On this concave ſurface ſeveral figures are ſuppoſed to be deſcribed, ſerving only as a means of referring to any known ſtar with greater facility: theſe figures are called conſtellations. It appears, that it is of no conſequence (as far as regards the apparent places of the ſtars) of what magnitude this imaginary ſphere is, always ſuppoſing the center of it to coincide with the center of view; thus in an artificial celeſtial globe, the angular poſition of the ſtars is ſuch, as if viewed from the center of the ſphere, would be ſimilar to that which is obſerved in the heavens.

It comes next in order to conſider of ſome properties of the ſphere immediately relating to Aſtronomy.

4. A circle, revolving round any diameter as an axis of motion, will generate a ſolid figure, which is called a ſphere.

5. The center of the generating circle is the center of the ſphere, and any right line paſſing through the center, and terminated by the ſurface of the ſphere, is called a diameter.

6. Any circle, the plane of which paſſes through the center of the ſphere, divides it into two equal parts, and is called a great circle, being diſtinguiſhed thereby from other circles, the planes of which not paſſing through the center, divide the ſphere unequally, and are called leſſer circles.

The angular diſtance of two points ſituated on the ſurface of the ſphere is meaſured by the arc of a great circle intercepted between them.

7. A line drawn from the interſection of any two lines in a plane, and perpendicular to them, is perpendicular to the plane itſelf.

Hence any two lines in a plane, which are inclined to each other, being parallel to a given plane, the two planes will be parallel.

Thus if any two lines inclined to each other in a plane be horizontal, the plane will be parallel to the horizon.

8. A diameter of a ſphere, drawn perpendicular to the plane of any great circle, is called the axis, and the extremities of the axis are called poles of the great circle.

1. Every point in the circumference of a great circle is 90° diſtant from the poles.

2. Any two great circles of a ſphere biſſect each other, becauſe the interſection of their planes is a diameter common to them both, and every circle is biſſected by its diameter.

9. Great circles paſſing through the poles of a great circle are called its ſecondaries.

Any ſecondary biſſects its great circle and all the leſſer circles parallel to it.

10. The inclination of two planes to each other, is the ſame as the inclination of

two lines drawn from the ſame point in the common interſection of the planes, and perpendicular to it.

11. The inclination of the planes of two great circles to each other, is meaſured by the arc of a ſecondary common to them both, and intercepted between the planes.

12. The planes of ſecondaries are perpendicular to the planes of their great circles, becauſe the inclination of a ſecondary to its great circle is meaſured by a quadrant.

13. The inclination of the circumferences of two great circles is called a ſpherical angle, and is the ſame as the inclination of the planes of the circles.

The meaſure of a ſpherical angle is, therefore, an arc of a ſecondary, common to both the great circles and intercepted between them.

14. Three arcs of a great circle, neither being greater than 180°, compoſe a ſpherical triangle, in which there are ſix quantities, viz. three ſides and three angles, any three of which being given, the reſt may be determined by the rules in Trigonometry.

As the angular poſition of objects ſeen from a given point, and ſituated in the ſame plane, is meaſured upon the circumference of a circle of any indeterminate radius, the center of which coincides with the center of view, in like manner the angular poſition of objects ſituated in different planes is eſtimated, by referring them to the ſurface of a ſphere, the center of which coincides with the center of view.

In what manner the properties of the ſphere above deſcribed are applied to the determination of the angular poſition and motion of the heavenly bodies will be next conſidered.

CONCERNING THE APPPARENT PLACE AND MOTION OF THE HEAVENLY BODIES.

1. All the ſtars appear to revolve daily in the heavens, deſcribing circles the planes of which are parallel to each other.

2. Among the circles deſcribed by the heavenly bodies in their diurnal revolution, there is one only the plane of which paſſes through the ſpectator's center of view.—This circle is called the equator.

The eye of the obſerver is ſuppoſed to be coincident with the center of the ſphere, upon the concave ſurface of which all the heavenly bodies are diſpoſed; the equator therefore muſt be a great circle of the heavens.

All other circles deſcribed by ſtars ſituated out of the equator, are leſſer circles of the ſphere parallel to the equator, and are called parallel's of declination.

3. If one extremity of a line continuing fixed, the other be always directed to a ſtar revolving in a leſſer circle, the line will by its motion deſcribe the ſurface of a cone, the vertex of which coincides with the fixed extremity of the line abovementioned, and the circumference of the baſe with the leſſer circle, which the ſtar deſcribes parallel to the equator.

4. Two points taken 90° from the equator, are called the poles of the equator, or poles of the world.

5. Secondaries to the equator are called circles of declination; of theſe, 24 which divide the equator into equal parts, containing 15° each, are called hour circles.

6. The declination of a ſtar is its angular diſtance from the equator; meaſured on a circle of declination paſſing through the ſtar.

Thus, in the middle of ſummer, the ſun's declination is 23°. 28′ north; when the ſun is in the equator his declination is 0.

7. Stars appear to move with the leaſt velocity which are the moſt remote from the equator, where the velocity of their apparent motion is the greateſt.

The apparent velocity of a ſtar in its diurnal revolution is as the coſine of declination.

8. The ſun and ſome particular ſtars, hereafter to be mentioned, are obſerved to change their places among the fixed ſtars, and to perform their revolutions in various periodic times.

The fixed ſtars always preſerve the ſame angular diſtances from each other, whereby they are diſtinguiſhed from the ſun and planets, which continually change their places both with regard to each other and the fixed ſtars.

The proper motions which have been diſcovered in the ſtars, Arcturus, Capella and ſome others, are not here conſidered.

9. The ſun compleats his revolution among the fixed ſtars in ~~366~~ days, 6 hours, 365 and about 9 minutes, deſcribing a great circle in the heavens, called the ecliptic.

10. The ecliptic is obſerved to be inclined to the equator at an angle of about 23° 28′.

The ecliptic and equator, being great circles, biſect each other; wherefore the points of interſection muſt be 180 degrees diſtant; theſe are called the equinoctial points, for reaſons hereafter to be given.

11. The ecliptic is divided into twelve portions, called ſigns, each containing 30 degrees; the firſt point of Aries coincides with one of the equinoctial points, and the firſt of Libra with the other.

The twelve ſigns are as follow.—Aries, Taurus, Gemini,
♈ ♉ ♊
Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capri-
♋ ♌ ♍ ♎ ♏ ♐ ♑
cornus, Aquarius, Piſces.
♒ ♓

The apparent motion of bodies in the ſame direction as the ſun, appears to move among the fixed ſtars, is called *motus in conſequentia*; and the motion in the contrary direction, *motus in antecedentia*.

12. The right aſcenſion of a ſtar is meaſured by an arc of the equator, intercepted between the firſt point of Aries, and a circle of declination paſſing thro' the ſtar, reckoning according to the order of the ſigns.

Thus in the middle of ſummer the ſun's right aſcenſion is 90 degrees.

13. The longitude of a ſtar is meaſured by an arc of the ecliptic, intercepted be-

tween the firſt point of Aries, and a ſecondary to the ecliptic paſſing through the ſtar, reckoning according to the order of the ſigns.

Thus in the middle of ſummer the ſun's longitude is 90° when the ſun enters libra the longitude is = 180°

14. The latitude of a ſtar is its angular diſtance from the ecliptic, meaſured on a ſecondary to the ecliptic paſſing thro' the ſtar.

The latitude of the ſun is therefore nothing.

15. A ſecondary, common to the ecliptic and the equator, is called the ſolſtitial colure: a ſecondary to the equator, paſſing through the equinoctial points, is called the equinoctial colure.

The inclination of the ecliptic to the equator is therefore meaſured by an arc of the ſolſtitial colure intercepted between them, and this arc will be the meaſure of the ſun's greateſt declination = 23°. 28′.

16. The horizon is a great circle in the heavens, the plane of which is perpendicular to a plumb line hanging freely and at reſt.

It is evident that the horizon with all the circles depending on it are moveable; every different ſpectator having a different horizon.

The pole of the horiſon above us is called the Zenith, the other pole the Nadir.

17. Secondaries to the horizon are called vertical circles.

The altitude of a ſtar is meaſured by the arc of a vertical circle, intercepted between the ſtar and the horizon; the complement of the altitude to 90° is the ſtars zenith diſtance.

18. A ſecondary, common to the equator and the horizon, is called the meridian; the meridian, therefore, paſſes through the pole of the equator and the zenith.

Hence the meridian is a vertical circle, and the inclination of the equator to the horizon is meaſured by an arc of the meridian intercepted between them.

19. The azimuth of a ſtar is meaſured by an arc of the horizon, intercepted between the meridian and a vertical circle which paſſes through the ſtar.

20. A vertical circle, which cuts the meridian at right angles, is called the prime vertical.

The interſections of the meridian and prime vertical, with the horizon are called the cardinal points; or the north, eaſt, weſt, and ſouth; the whole horizon being divided into 32 equal parts forms the mariner's compaſs.

21. The diurnal apparent motion of the ſtars is from eaſt to weſt.

When a ſtar is ſaid to move apparently from eaſt to weſt, it is meant that a vertical circle always paſſing through the ſtar continually changes its azimuth, leaving the eaſtward and approaching the weſtward parts of the horizon.

22. The direction in which the ſun moves in the ecliptic, among the fixed ſtars, is contrary to that of his apparent diurnal motion. The direction of the planets motion among the fixed ſtars is moſt frequently obſerved to be *in conſequentia*, but ſometimes *in antecedentia*: the planets are alſo obſerved at times to be ſtationary.

23. An arc of the meridian, intercepted between the zenith and the equator, is called the latitude of the place.

24. An arc of the equator, intercepted between the meridians of two places, is called the difference of their longitude; or, if the longitude of one of them be referred to the meridian of the other, the arc above deſcribed is called the longitude.

Thus if the longitude of Rome be referred to the meridian of London, it is $9^{\circ}\frac{3}{4}$ eaſt.

When the center of the ſun is upon the meridian of any place, it is there ſaid to be noon-day; which therefore will be at different times at different places; and ſince the time elapſed between the ſun's leaving and returning to the ſame meridian is 24 hours, during which a circle of declination paſſing through the ſun's center will have deſcribed 360°, eſtimated on the equator; it follows that in one hour it will have deſcribed 15°; in one minute of time, 15′; and in one ſecond of time, 15″ of a degree, &c.

25. The tropics are two parallels of declination, the diſtance of which from the equator is equal to the ſun's greateſt declination.

The diſtance of the tropics from the equator is therefore =23°. 28′.

26. The arctic and antarctic circles are two parallels of declination, the diſtance of which from the poles is equal to the diſtance of the tropics from the equator.

The arctic circle is 23°. 28′. from the north pole, and the antarctic at the ſame diſtance from the ſouth.

27. The apparent time of day is meaſured by the angle contained between the meridian of the place and a circle of declination paſſing through the ſun's center, allowing one hour for 15°, and proportionally for minutes and ſeconds of a degree.

The arc of any great circle of the ſphere will be determined, (that is the proportion of it to the circumference will be known,) if the angle ſubtended by its extremities at the center of the ſphere be meaſured; in order therefore to apply the preceding principles to practical Aſtronomy, it will be neceſſary to premiſe a few propoſitions concerning the menſuration of angles.

CONCERNING THE MENSURATION OF ANGLES.

1. IF two objects appear coincident with the extremities of a circular arc to the eye of a ſpectator, ſituated at the center of the circle, that arc will meaſure the obſerved angle ſubtended by the objects.

1. Let a repreſent any arc, A° the angle ſubtended by it, p a right angle, c the circumference, and r the radius of the circle; we have the following proportion:

$A^\circ : 4p :: a : c$, wherefore $A^\circ = \frac{4p \times a}{c}$; and ſince $4p$ is the ſame in all circles, A° will be proportional to $\frac{a}{c}$ or $\frac{a}{r}$: that is, the angle will be as the arc directly, and the radius of the circle inverſely.

2. Into whatever number of equal parts the circumference of a circle is divided, a right angle will be meaſured by $\frac{1}{4}$ of them; wherefore if the circumference be divided into 360 equal parts or degrees, a right angle will be ſubtended by 90°.

3. The arcs which meaſure ſmall angles are nearly equal to their ſines, chords or tangents, the radius of the circle being given: ſmall angles will therefore be nearly proportional to their ſines, &c. in the ſame circle, and in different circles will be as their ſines directly, and the radii of the circles inverſely.

4. Small angles ſubtended by a given rectilineal object, which is perpendicular the axis of viſion, will be greater in the ſame proportion as the diſtances of the obſerver are leſs.

5. Let $M^\circ = 57^\circ.\ 17'.\ 45''.$ or $206265''$ (being the angle which is ſubtended by an arc $=$ radius.)—Since it appeared by note 1, that $A^\circ. = \frac{4p \times a}{c}$ and by the properties of the circle $\frac{4p}{c} = \frac{M^\circ}{r}$, it follows that $A^\circ = \frac{a}{r} \times M^\circ$. Hence $a = \frac{A^\circ}{M^\circ} \times r$.

6. If s be the ſine, chord, or tangent of a ſmall angle, the angle will be nearly $= \frac{s}{r} \times M^\circ$.

7. If $A^\circ =$ an angle of $1''$, ſince $M^\circ = 206265''$, it follows that $\frac{A^\circ}{M^\circ} = \frac{1}{206265}$; and an arc ſubtending an angle of $1''$ $= \frac{r}{206265}$, that is $\frac{1}{206265}$ part of radius: an arc of $1'$ therefore $= \frac{60r}{200265}, = \frac{1}{3437\ 8}$, or about $\frac{1}{3438}$ part of radius.

By the preceding rules any two of theſe quantities, viz. a circular arc, the angle ſubtended by it, and the radius of the circle being known, the other may be determined.

The arcs of quadrants and ſectors uſed for the menſuration of angles are divided into degrees, &c. and in inſtruments of a very large radius the diviſions are extended (by means of a nonius hereafter deſcribed) to ſeconds of a degree.—An index of equal curvature with the arc is applied to the extremity of a moveable radius, by means of which the angle ſubtended between two obſerved objects is determined.

To the moveable radius above-mentioned, a teleſcope is applied, having a fixed point marked in the center of the field; a line joining this point and the center of the object glaſs is called the line of collimation: this line determines the directtion in which an obſervation is made.

2. An obſerved angle will err from the truth, if the line of collimation be inclined to the plane of the quadrant with which the obſervation is taken.

Suppoſe the quadrant to be fixed in any given plane, the line of collimation may be conſidered as the radius of a ſphere, a great circle of which coincides with the arc of the quadrant.

If the line of collimation be inclined to the plane of the quadrant, it will, during the motion of the index, be continually directed to a leſſer circle of the ſphere, and therefore cannot determine the angular diſtance of objects, which is always meaſured upon a great circle.

It is ſuppoſed that every part of the graduated arc, and the index contiguous to it, lies in the ſame plane, which is called the plane of the quadrant, and is, perpendicular to the axis of motion upon which the quadrant or the index applied to it, moves.

3. The plane of a quadrant being vertical, if one of the radii containing the right angle be horizontal, the other will be perpendicular to the horizon: an intermediate radius being directed to any ſtar, &c. will divide the quadrant into two arcs, one of which meaſures the zenith diſtance, and the other the ſtars altitude above the horizon.

Although the plane of the quadrant be ~~verticle~~ Vertical and coincident with that in which the line of collimation moves, yet the following errors may cauſe a difference between the true and obſerved altitudes.

1. If the line of collimation be not horizontal when the index points to 0 (or 90°.)

2. If the diviſions of the quadrant are inaccurate.

3. If the diviſion pointed to by the index be read of erroneouſly.

The direction of all theſe errors coinciding with the plane of the quadrant ſhould be conſidered together. The firſt circumſtance which occurs is, that they do not neceſſarily combine to increaſe or diminiſh the obſerved altitude; but it may happen that their effects being contrary, ſhall correct, or or even deſtroy each other.

In order to form an eſtimate of the error which will probably ariſe on this account, let us ſuppoſe, firſt that the diviſion of the quadrant is mathematically exact; and that the adjuſtment of the line of colimation to the point o, and the reading off can be depended upon to d'', and to no greater accuracy.— It is an even probability which of the ſeconds between $\pm d''$ and is equal to the error on each account; for example the ſum of the two errors may $\pm$ be $2d''$, $\pm \overline{2d-1}''$, $\pm \overline{2d-2}''$, &c. of which combinations there are $4d^2$, among theſe there are $2d^2$, and no more, each of which exceeds $\frac{d'' \times \sqrt{2}}{1+\sqrt{2}}$ (d being a very large number) from whence it is concluded that the probable error, or that which is an even chance the error of the obſervation does not exceed $= \frac{d'' \times \sqrt{2}}{1+\sqrt{2}} = \frac{17\,d''}{29}$, nearly. If $d'' = 60''$ the probable error $= \frac{17 \times 60''}{29} =$ about $35''$.

It appears likewiſe that if the error in the diviſion of the quadrant be taken into conſideration with the other two, and the limit of it be of equal magnitude with the former, viz. d'', the error of an obſervation compounded of the three errors above deſcribed, may be either $\pm 3d''$, $\pm \overline{3d-1}''$, or $\overline{3d-2}''$, &c. of which combination there are $8d^3$, among theſe there are $4d^3$ and no more, each of which exceeds $\frac{d''}{1.414}$ (d being a very large number) from whence it is inferred, that the probable error upon the whole $= \frac{d''}{1.414}$: If $d'' = 60''$; the probable error $= 42''$ nearly.

4. Although an aſtronomical quadrant be free from all the errors above deſcribed, if the plane of it ſhould be inclined to that of the vertical circle in which an obſerved ſtar is ſituated, the altitude deduced from obſervation will be greater than the true altitude.

If the deviation of the quadrants's plane from that of the vertical circle be an angle (incomparably ſmaller than the altitude) the verſed ſine which $= u$, let the ſine of the obſerved altitude $= s$, the coſine $= c$, radius $= 1$, $M^\circ = 57^\circ.\ 17'.\ 45''$:

then the difference of the obſerved and true altitudes, ariſing from the inclination of the quadrant to the vertical circle, $= scu \times M^{\circ}$.

Cor. I. The deviation of the quadrant being given, the error $scu \times M^{\circ}$ will be the greateſt when the altitude $= 45^{\circ}$, becauſe the product of the ſine and coſine of 45° is greater than the product of the ſine and coſine of any other angle.

Cor. II. If the altitude be the ſame, the error in obſerving it will be in a duplicate ratio of the deviation of the quadrant's plane from the vertical, becauſe the verſed ſines of arcs are in a duplicate ratio of the arcs themſelves, when they are diminiſhed *ſine limite*.

The obſerved altitude being a ſmall angle $= d''$, the condition of the propoſition is altered when the error of the quadrant bears a conſiderable proportion to d''.—If u repreſents the verſed ſine of the quadrant's error as before, the error in altitude will now $= ud''$.

5. If the line of collimation be inclined to the plane of a quadrant otherwiſe accurately adjuſted, the altitude of a ſtar deduced from an obſervation taken by it will be greater than the true altitude.

If the inclination of the line of collimation to the plane of the quadrant be an angle (incomparably ſmaller than the zenith diſtance) the verſed ſine of which $= u$, let $T =$ the tangent of the obſerved altitude, $M^{\circ} = 57^{\circ}.\ 17'.\ 45''$, the difference between the true and obſerved altitudes $= uT \times M^{\circ}$.

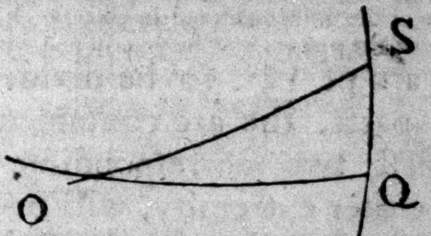
Cor. I. The altitude being given, the error in obſerving it will be in a duplicate ratio of the error of the line of collimation.

Cor. II. The error of the line of collimation being given, the difference between the obſerved and true altitude will be as the tangent of the altitude.

When the obſerved zenith diſtance is ſo ſmall that the error of the line of collimation bears a conſiderable proportion to it, let $e =$ the error of the line of collimation, $d =$ the ſmall zenith diſtance, deduced from ~~direction,~~ Observation the error in altitude now becomes $= \sqrt{d^2 + e^2} - d$.

6. Suppoſe a great circle to paſs through a ſtar, and to interſect the horizon at any angle; if this angle and the arc of the great circle intercepted between the ſtar the horizon be taken by obſervation, the ſtars altitude above the horizon will be determined.

This indirect method of meaſuring the altitudes of objects is attended with conſiderable advantages.



Let S repreſent the place of a ſtar, SQ the arc of a vertical circle paſſing through the ſtar, OQ an arc of the horizon, SO the arc of a great circle paſſing through the ſtar and interſecting the horizon at the angle SOQ. According to the above method, the arc OS and the angle SOQ are obſerved in order to find the arc SQ.—Suppoſe each of theſe obſervations, i. e. of OS and SOQ liable to an error $= d''$, then the correſponding difference in the arc $SQ = \frac{d'' \times \text{tang. } SQ}{\text{tang. } OS} + \frac{d'' \times \text{tang. } SQ}{\text{tang. } O}$; which quantity (if d'' be given) is the leaſt (SQ being leſs than 30°) or the greateſt poſſible (SQ being greater than 30°) when the ſine of the angle O is a mean proportional between the ſine of SQ and radius; that is, when the arc SO is equal to the meaſure of the angle SOQ.—In either caſe it is found that, ſuppoſing the ſine of $SQ = s$, radius $= r$, the quantity $\frac{d'' \times \text{tang. } SQ}{\text{tang. } OS} + \frac{d'' \times \text{tang. } SQ}{\text{tang. } O}$ will be equal to $2d'' \times \sqrt{\frac{s}{r+s}}$: but ſince the two errors abovementioned do not neceſſarily combine either to increaſe or diminiſh the altitude, and may correct or even deſtroy each other, it appears from theſe conſiderations, and from the note to art 3. that the probable error, or that which it is an even chance, the error of the obſervation ſhall not exceed, $= \frac{d'' \times \sqrt{2}}{1+\sqrt{2}} \times \sqrt{\frac{s}{r+s}} = \frac{17d''}{29} \times \sqrt{\frac{s}{r+s}}$ nearly: whereas the probable error ariſing in the direct menſuration of the vertical arc $SQ = \frac{d''}{2}$, which is in all caſes greater than $\frac{17d''}{29} \times \sqrt{\frac{s}{r+s}}$.—In determining the ſmaller altitudes by the indirect method, the probable error will become very inconſiderable.—For inſtance, in meaſuring an altitude of 1°, if the obſervations of OS and SOQ are each liable to an error, which may ariſe to, but not exceed 60″, the probable error in the altitude $= \frac{17 \times 60''}{29} \times \sqrt{\frac{\text{ſine } 1^\circ}{r + \text{ſine } 1^\circ}}$, which is no more than 4″.6, whereas the probable error which would be incident to the direct obſervation of the ſame altitude is 30″.

Theſe principles may be applied to the conſtruction of a micrometer, from the theory of which it appears, that making all allowance for the unavoidable imperfections in the conſtruction of the inſtrument and in obſervation, the apparent diameter of the planets may be determined within 1″.

7. If an arc of 15°. 30′ be divided into 30 equal parts, the arc contained between the firſt two correſponding diviſions from either extremity, will be 1′, the arc between the next two diviſions will be 2′, between the next 3′, and ſo on, till the arc contained between the laſt correſponding diviſions = 30′, or ½ a degree.

In general, if an arc or right line containing $n+1$ parts, each of which $= p$, be divided into n equal parts, the diſtance between the firſt two correſponding diviſions from coincidence $= \frac{p}{n}$, between the two next $\frac{2p}{n}$, and ſo on till the diſtance between the laſt $= \frac{np}{n}$, or p.

The nonius applied to aſtronomical and other inſtruments is conſtructed upon theſe principles.

By means of this contrivance, angles are obſerved to an accuracy, much beyond what could be effected by actual diviſion of the quadrant.

Since an arc which ſubtends a ſecond of a degree $= \frac{1}{206265}$ part of radius, it is eaſy to find the length of any arc contained between two contiguous diviſions of a quadrant; thus if the radius of the quadrant be 3 inches, and if the arc be divided into ½ minutes, the ſpace contained between two contiguous diviſions $= \frac{1 \times 30 \times 3}{206265}$, or $\frac{1}{2292}$ parts of an inch.

If the radius of the earth be 3970 miles, or 3970 × 1760 yards, it follows that a ſecond of a degree meaſured upon a great circle of the earth $= \frac{3970 \times 1760}{206265} = 34$ yards nearly.

If ſeveral obſervations be taken in order to determine the angle ſubtended between two given objects, the reſults of theſe obſervations will probably differ ſomething from each other: It is uſual in this caſe to add the reſults together, and to divide the ſum by the number of them: the quotient is called the mean of the whole, and every thing elſe being the ſame, will approach nearer to the truth, the greater number of obſervations are taken, from which the mean is derived--thus, if p, q, and r be the reſults of 3 obſervations, the mean of the three will be $\frac{p+q+r}{3}$.

PRACTICAL INFERENCES DEDUCED FROM THE PRECEDING PRINCIPLES.

1. Half the difference between the greateſt and leaſt meridian altitudes of the ſun's center, taken in any given latitude, is equal to the inclination of the ecliptic to the equator.

The apparent altitude of all the heavenly bodies being increaſed by refraction, is allowed for by referring to tables which are conſtructed for that purpoſe.

The obſervation of a ſtar's altitude, implies the previous determination of the horizon—this is effected two ways. 1. By a plumb line hanging freely and at reſt: a plane perpendicular to this line muſt be horizontal. 2. By the level.

[illegible] a ſtar be obſerved by reflexion from an horizontal poliſhed ſurface, the angle ſubtended by the image and the object, at the eye of the ſpectator, will be double to the ſtar's altitude above the horizon.

In order to find the altitude of the ſun's center, two methods are uſed; 1. By obſerving the altitudes of the higheſt and loweſt points of the diſk: half the ſum will be the altitude of the ſun's center. 2. By obſerving the altitude of the loweſt point only, and adding the ſun's apparent ſemidiameter, correſponding to the time of obſervation, taken from an ephemeris, wherein it is previouſly calculated.

2. The latitude of a place, is equal to the altitude of the pole of the equator above the horizon.

1. The diſtance between the zenith and the horizon, is equal to the diſtance between the pole and the equator, each of them being 90°: take away the diſtance of the zenith from the pole, which is common to both, and there remains the diſtance of the pole from the horizon, equal to the diſtance of the zenith from the equator, or the latitude of the place.

2. The diſtance of the zenith from the pole, is equal to the complement of the latitude to 90°.

3. Hence is derived the menſuration of the earth's circumference. It appears by experiment, to an obſerver, who proceeds directly on the meridian 69.3 miles in the latitude of 57°, that the difference of the pole's altitudes is 1°. From whence it is inferred, ſuppoſing the earth to be perfectly ſpherical, that an arc of one degree on the earth's ſurface = 69.3 miles, and conſequently that the earth's circumference = 69.3 × 360 = 24948, and the diameter = 7941 miles.

The ſperoidical figure of the earth, cauſes a ſmall difference in the length of a degree, when meaſured on a great circle of the earth in different latitudes.

3. If the greateſt and leaſt altitudes of a circumpolar ſtar be determined by obſervation, half the ſum will be the latitude of the place.

The ſtar uſually called the pole ſtar, is not exactly on the pole of the equator, the above method is therefore (among others) made uſe of in order to determine the true altitude of the pole.

4. The inclination of the equator to the horizon, is equal to the complement of the latitude to 90°.

The diſtance of the zenith from the horizon = 90°. The meridian diſtance of the zenith from the equator = the latitude: the arc of the meridian therefore intercepted between the equator and the horizon muſt be = the complement of the latitude to 90°.—This arc meaſures the inclination of the equator to the horizon, becauſe the meridian is a ſecondary common to both thoſe circles.

5. If equal altitudes of a ſtar be taken on different ſides of the meridian, the time elapſed between the obſervations being biſſected, will give the inſtant when the ſtar was on the meridian.

From hence is derived the uſual method of determining the meridian by means of a tranſit teleſcope (previouſly adjuſted to the zenith) and a well regulated clock.

If the inſtant of the ſtar's tranſit obſerved in the teleſcope, does not biſſect the time elapſed between the obſervations of the equal altitudes, the poſition of the tranſit muſt be corrected.

The poſition of the meridian teleſcope, previouſly adjuſted to the zenith, may be verified by obſerving the the tranſits of a circumpolar ſtar, paſſing above and below the pole: if the time elapſed between the tranſits be not exactly 11*h*. 58*m*. 2s being half of the time in which a fixed ſtar compleats its diurnal revolution, the line of collimation does not move in the plane of the meridian.

6. The declination of a ſtar, is the difference of the ſtar's meridian zenith diſtance, and the latitude of the place.

The latitude of a place being known, the declinations of the ſtars are determined from the obſervation of their meridian zenith diſtances.

7. The difference in right aſcenſion of two ſtars eſtimated in time, is equal to the time elapſed between their tranſits over a given meridian.

From hence we may verify, the poſition of a tranſit teleſcope, or aſtronomical quadrant, previouſly adjuſted to the zenith, i.e. we may aſcertain whether the line of collimation moves in the plane of the meridian, by obſerving the tranſits of two ſtars, the declinations of which are conſiderably different: if the time elapſed between the tranſits be equal to the difference of the ſtars right aſcenſions, the line of collimation moves in the plane of the meridian, if not, a correction is neceſſary.

This is the method uſed by the Rev. Mr. Maſkelyne, in his experiment upon the attraction of the mountain Schehallien, deſcribed in the Phil. Tranſact. for 1776.

The declination of any ſtar, and the inclination of the ecliptic to the equator being given, the right aſcenſion is determined by the ſolution of a right angled triangle.

The right aſcenſions of the reſt of the ſtars are found by the obſervation of their tranſits over the meridian.

8. The latitude of a place is equal to the ſun's meridian zenith diſtance + the declination in ſummer, and — the declination in winter.

Hence is deduced the uſual method of determining the latitude of a place, by obſerving the ſun's meridian altitude or zenith diſtance. The ſun's altitude or zenith diſtance being accurately meaſured, the latitude will be deduced very near the truth. For if the ſun's place in the ecliptic be computed true within 20″, the error of the declination cannot exceed 7″.9: but will never be ſo much except in the extreme caſe, when the error of the ſun's longitude is the greateſt poſſible, viz. 20″, and the ſun is near either of the equinoctial points: we may conclude in general, that the computation of the ſun's declination does not err more than 2″ or 3″ from the truth.—The meridian altitude and declination of a celeſtial object being determined, and proper allowance being made for the effects of refraction, &c. the latitude of a place may be determined, as will appear in the following example, of an obſervation taken at Cambridge, Auguſt the 7th, 1776.

Apparent meridian Altitude of the ſun's lower limb — —— = 53°. 46′. 8″
Sun's apparent ſemidiameter = - - - - 0°. 15′. 50″

Apparent altitude of the ſun's center = - 54°. 1′. 58″
Subtract on account of refraction = - - 0°. 0′. 41″

Altitude of the ſun's center = - - - - 54°. 1′. 17″
Zenith diſtance of the ſun's center = - - 35°. 58′. 43″
Add the ſun's declination = - - - - - 16°. 13′. 57″

The latitude of Cambridge = - - - - 52°. 12′. 40″

When the utmoſt accuracy is required, the effects of the temperature and weight of the atmoſphere, and the ſun's parallax muſt be taken into the computation; but the variations in the obſerved altitude produced from theſe cauſes are too minute to be here particularly conſidered.

9. The latitude of a place, and declination of the ſun being given, the apparent time may be determined, from an obſervation of the ſun's altitude.

The diſtance of the zenith from the pole, the complement of the ſun's declination to 90°, and the ſun's zenith diſtance, are the ſides of a ſpherical triangle, one of the angles of which is that contained between the meridian and declination circle paſſing through the ſun's center, and is the meaſure of the apparent time, which is therefore determined by the ſolution of the triangle.

This is the method of finding apparent time, uſually practiſed at ſea—the ſame problem may be effected by obſerving the altitude of a known fixed ſtar, with very little variation in the computation: but in each caſe, ſince it is impoſſible to meaſure altitudes above the horizon abſolutely free from error, the obſervation muſt be taken in ſuch a manner, that a given variation in the obſerved altitude will cauſe the leaſt poſſible variation in the time deduced: which is done by taking the altitude of the ſun or ſtar when on the prime vertical.—Let $\dot{a}$= the given variation of a ſtar's altitude, $\dot{b}$ = the correſponding variation of the hour angle, then we ſhall have $\dot{b} = \dot{a} \times \frac{\overline{\text{radius}}^{2}}{\text{Cos: latitude} \times \text{ſine azimuth}}$. Therefore, ſince at a given place $\frac{\overline{\text{rad.}}^{2}}{\text{coſ. lat.}}$ is conſtant, and $\dot{a}$ being by the ſuppoſition invariable; $\dot{b}$, or the variation of the hour angle deduced from obſervation will be proportional to $\frac{1}{\text{ſine azimuth}}$, which quantity is the leaſt when the azimuth = 90°, or when the ſun or ſtar is ſituated on the prime vertical.

Cor. I. It appears from hence that (every thing elſe being the ſame) it is of no conſequence to the accuracy of the time deduced, at what altitude a ſtar is obſerved upon the prime vertical.

Cor. II. In the latitude of 52°.12′.40″, an error of 1′ in obſerving the altitude of a ſtar upon the prime vertical cauſes an error of $\frac{1'}{\text{coſ. } 52^\circ.12'.40''} = 97''.9$ = ſix ſeconds and a half in the time deduced.

If the obſervation be taken on any other vertical, the error in altitude being the ſame, the error in time before found (6 ½ ſeconds) will be increaſed in the ratio of the ſine of the ſtars azimuth to radius.

10. If the difference of the apparent time of day at any two places be known, (at the ſame inſtant) their difference of longitude will be known.

One of the methods uſed for determining the difference of longitude, is by two perſons obſerving an eclipſe of the ſun, one of Jupiter's ſatellites, &c. or the occultation of a fixed ſtar by the moon.—If either of theſe phenomena happened at different apparent times to the two obſervers, the difference of theſe times (allowing for the effects of parallax, if neceſſary) will give the difference of longitude required. This method can be practiſed only on land. The following methods are uſed at ſea: --a watch well regulated to time at a given place, being carried to ſea eaſtward or weſtward, will ſhew apparent time (by means of adding or ſubtracting the equation) different from that which is deduced from obſervation of the ſun or ſtars. This difference of time is eaſily converted into the difference of longitude required.---The difference of longitude is now uſually determined at ſea by obſerving the angle ſubtended by the moon's center and a fixed ſtar, not very diſtant from the moon's orbit.

The moon's diſtances from ſeveral of the fixed ſtars, and the times correſponding, are calculated in the Nautical Ephemeris for the meridian of Greenwich; hence by obſerving the diſtance of the moon from a fixed ſtar, the time at Greenwich may be determined, and the time at the place of obſervation being known, the difference converted into degrees and minutes, will give the longitude from Greenwich.

But it is here neceſſary to obtain the true diſtance of the moon from the ſtar, free from the effects of parallax and refraction, which altering the apparent altitudes of the ſtar and moon, cauſes a variation in their apparent diſtance. The apparent altitudes of the moon's center and a ſtar, with their apparent diſtance from each other being given by obſervation, we obtain the true difference of their azimuths, which is not affected by refraction or parallax. The apparent zenith diſtances of the moon and ſtar being known, their elevations ariſing from refraction, as well as the depreſſion of the moon's center cauſed by parallax, will be known from the tables, from whence we derive their true zenith diſtances: and the difference of the azimuths being previouſly determined, the ſolution of a ſperical triangle, gives the true diſtance of the moon's center from the ſtar.--- This method of aſcertaining the longitude at ſea, was firſt brought into uſe by the Rev. Mr. Maſkelyne.

11. In a given latitude no ſtar will ſet below the horizon, the polar diſtance of which is leſs than the latitude of the place.

12. In latitudes leſs than about 66°. 32′, the ſun's center can never be totally above or below the horizon during 24 hours.

13. When the ſun's declination is greater than the diſtance of the zenith from the pole, the ſun's center will not ſet below the horizon, or will not riſe above it, according as the declination is of the ſame, or a contrary denomination to the latitude.

In this and the preceding article, the effects of refraction are not conſidered.

14. If an obſerver be ſituated under the equator, the plane of his horizon is perpendicular to the plane of the equator and parallels of declination.

The Sphere of the heavens, in this poſition, with regard to the ſpectator's horizon, is denominated a right ſphere.—The equator, and all the parallels of declination in a right ſphere, are biſſected by the horizon; therefore the ſun and all the ſtars are above the horizon during one half of their diurnal revolution.

The poles of the world in a right ſphere coincide with the horizon.

15. If an obſerver be ſituated at either pole, his horizon is coincident with the equator, and the parallels of declination are parallels to the horizon.

The ſphere of the heavens in this poſition, reſpecting the ſpectators horizon, is called a parallel ſphere. The ſun, when above the horizon, appears to revove in a leſſer circe, parallel to the horizon, and at an altitude above it equal to his declination.

16. In all other poſitions of the ſphere of the heavens, except thoſe deſcribed, the horizon is inclined to the equator and its parallels at ſome angle leſs than 90°: in this caſe, the poſition of the ſphere is ſaid to be oblique.

If a tangent be drawn to the ſurfaces of the ſun and earth, in the ſame plane with the line joining their centers, a circle deſcribed through the point of contact at the earth's ſurface, the plane of it being perpendicular to the abovet-mentioned line, will be the circle which terminates light and darkneſs.

The plane of this circle nearly, though not exactly, paſſes through the earth's center: it may be conſidered as a great circle, except when the utmoſt accuracy is required.

17. When the ſun is in either of the equinoctial points, the circle bounding light and darkneſs coincides with the poles of the world, and conſequently biſſects all the parallels to the equator.

Hence it follows that, when the ſun is is in either of the equinoctial points, the days and nights in all parts of the world will be equal.

18. When the ſun's declination is north, the circle terminating light and darkneſs divides the northern parallels of latitude unequally, thoſe parts being the greateſt which lie within the ſaid circle.

Hence the days will be longer than the nights to the inhabitants of the northern parallels of latitude, when the ſun's declination is north.

19. When the ſun's declination is ſouth, the circle terminating light and dark-

neſs divides the northern parallels of latitude unequally, that part of them being the greateſt, which lies without the circle of illumination.

Hence it is, that the nights are longer than the days, to the inhabitants of the northern parallels, when the ſun is ſouth of the equator.

20. When the ſun's declination is greater than the complement of latitude to 90°, the parallel of latitude paſſing through the place, will fall either entirely within, or entirely without the circle of illumination.

The ſun's center will therefore appear either to be entirely above or below the horizon to the inhabitants of that parallel, according as the declination is of the ſame, or of a contrary denomination to the latitude of the place.

ON DIALLING, AND CONCERNING THE USE OF THE EQUATORIAL INSTRUMENT.

1. If rays of light proceeding from the ſun's center ſhine upon a right line parallel to the earth's axis, the ſhadow of the line will be in the ſame plane with the circle of declination, which paſſes through the center of the ſun.

The ſhadow of any line is a plane which paſſes through the line, and through the ſun's center: the ſhadow deſcribed in the propoſition, will cut the horizon, or any other great circle given in poſition, in the ſame right line, at a given time of the day, whatever be the ſun's declination.

2. If a line be applied to the center of a circle, the plane of which is given in poſition, and this line be adjuſted parallel to the earth's axis, the inclination of the line's ſhadow to the plane of the meridian, is the meaſure of the apparent

time of day; allowing one hour for 15° &c.

From hence it follows, that, if circles of declination be projected upon any plane given in poſition, and a line be applied to the plane, at their common interſection, parallel to the earth's axis, the ſhadow of the line will determine the apparent time of the day.

3. Let the axis of a ſphere, upon which the circles of declination are deſcribed, be directed to the pole of the heavens. The ſphere being biſſected by the plane of any great circle, the ſection will be a dial, of which the axis of the ſphere is called the ſtyle.

The interſection of the dial with the meridian is marked XII. the interſection of the adjacent hour circle, inclined to the meridian, at an angle of 15° is marked I. and ſo on; the intermediate times being marked at the interſections of the correſponding circles of declination, with the plane of the dial. Since, when the hour angle is the ſame, the ſection of the ſtyle's ſhadow is the ſame, whatever be the ſun's declination; it follows, that whenever the ſun ſhines upon the ſtyle, the time of the day being given, the ſhadow will always interſect the dial in the ſame right line: this line therefore will determine the apparent time.

A dial is called vertical, horizontal, equatorial, &c. according as the plane of it coincides with that of a vertical circle, the horizon or equator in the heavens.

4. The equatorial inſtrument conſiſts of 3 circles, the planes of which may be ſo adjuſted as to coincide with the planes of the horizon, equator, and any given circle of declination in the heavens reſpectively.

A teleſcope is applied to the declination circle, to the plane of which the line of collimation is parallel.

The moſt common uſes to which the equatorial inſtrument is applied, are as follow;

1. To determine the poſition of the meridian by a ſingle obſervation.
2. To find the apparent time of the day.
3. To direct the teleſcope to any point in the heavens, the

the right aſcenſion and declination of which are known. —In order to effect theſe problems, the inſtrument muſt be firſt rectified to the latitude of the place.

5. Let the equatorial circle be inclined to the horizon, at an angle = to the complement of the latitude to 90°.—The inſtrument is adjuſted to the latitude.

For the inclination of the equator in the heavens to the horizon, is equal to the complement of the latitude, to 90°.

6. In order to determine the meridian, the declination circle muſt be ſet to the declination of a known ſtar.—If the ſtar be brought into the centre of the field, by means of the equatorial and azimuth circles, the plane of the equatorial circle will coincide with that of the equator in the heavens, (ſetting aſide the effects of refraction.)

Hence the plane of a ſecondary to the equatorial and azimuth circles will coincide with the plane of the meridian; to which the line of collimation will be directed, when the equatorial index points to XII.

7. The inſtrument being adjuſted to the latitude, let the declination circle be ſet to the ſun's declination, correſponding to the time of obſervation:—if the line of collimation be directed to the ſun's center, by moving the equatorial and azimuth circles in their own planes, the arc of the equatorial circle intercepted between XII. and the point to which the index is directed, will determine the apparent time of the day.

In this caſe the equatorial circle becomes a dial, the plane of which is perpendicular to the ſtyle, or, as it is uſually called, an squatorial dial. It is to be obſerved, that, in both the preceding articles, the opparent elevation of the he celeſtial objects, above their true places, ariſing from refraction, will cauſe the poſition of the meridian, and the time deduced from obſervation, with the equatorial teleſcope,

cope, to deviate ſomething from the truth.—Theſe errors both in the hour angle and azimuth may be eſtimated thus.

Let $\dot{r}$ repreſent the apparent elevation of the ſtar, or ſun's center, cauſed by refraction: s, the angle contained between a vertical circle and a circle of declination, paſſing through the ſtar—then the variation of the hour angle, cauſed by the apparent elevation of the ſtar $= \frac{\dot{r}}{\text{coſ. lat.} \times \text{ſin. azimuth.}}$ and the cotemporary variation in the azimuth $= \frac{\dot{r} \times \text{Cotang. s.}}{\text{ſin. ſtars zenith diſt.}}$ Since by the properties of the equatorial inſtrument, when adjuſted, the plane of the declination circle will always be vertical when the equatorial index points to XII. it follows that, the poſition of the meridian found in art. 6, will be corrected by moving the azimuth circle, in a direction contrary to the ſun's azimuth, through an arc. $= \frac{\dot{r} \times \text{Cot. s.}}{\text{ſin. zen. diſt.}}$

It may be inferred from hence, that in the latitude 52° 12′ 40″, refraction will cauſe no error in the poſition of the meridian, determined as in art. 6, when the obſerved ſtar's declination and altitude = 62°. 45′.

8. Let the inſtrument be adjuſted to the latitude and the meridian; if the declination circle be ſet to the declination of any ſtar, &c. and the equatorial circle be moved in its own plane, the line of collimation will be always directed to the ſame parallel of declination, as the ſtar deſcribes in its diurnal revolution.

Hence it follows, that if the right aſcenſion and declination of any ſtar be given, the line of collimation of the equatorial teleſcope may be directed to it; by which means Jupiter, Venus, and the fixed ſtars of the firſt magnitude, may be obſerved in the day time.

9. The altitude of a celeſtial object above the horizon may be meaſured on the declination circle of the equatorial inſtrument.

When the equatorial index points to XII. the plane of the declination circle paſſes through the zenith, the line of collimation therefore being adjuſted horizontal, and afterward direction to a ſtar, by moving the declination and azimuth circles in their own planes; the index of the declination circle will determine the arc of the vertical circle, intercepted between the ſtar and the horizon.

10. The altitude of a celeſtial object may be determined, by meaſuring the arc of a great circle, intercepted between the ſtar and the horizon, and the inclination of the ſame great circle to the horizon.

This is the method deſcribed in page 92, art. 6. and may be put in practice by means of the equatorial inſtrument thus.—Let the ſine of the eſtimated altitude of the object, be s: elevate the equatorial circle above the horizon, to an angle, the ſine of which $=\sqrt{s}$, rad. being = 1. — The declination circle being ſet to 0, direct the line of collimation to the ſtar, by the equatorial and azimuth circles, moved in their own planes; obſerve the arc of the equatorial circle, intercepted between the index and VI. if the ſine of this arc = p. the ſine of the obſerved altitude will be equal to $\sqrt{s} \times p$. rad. being 1.

11. Suppoſe Q to repreſent the zenith, (vid. fig. in note to art. 6. p. 93.) s the place of a ſtar, Q s and Q o the arcs of two vertical circles; s o, a great circle paſſing through the ſtar, and equal to Q o.—If the arc Q o and the angle s o Q be determined by obſervation, the zenith diſtance s Q will be known.

This method of determining zenith diſtances is ſimilar to the indirect menſuration of altitudes before deſcribed, and is capable of great accuracy, when the arc to be meaſured is ſmall, eſpecially ſince the line of collimation by the conſtruction of the equatorial inſtrument, may be accurately adjuſted to the zenith.

The equatorial inſtrument may be applied to this method by means of the following rules—let the ſine of the eſtimated zenith diſtance = s: find an angle, the ſine of which $=\sqrt{s}$. radius being 1. Elevate the equatorial circle above the horizon, to an angle, the ſine of which $=\sqrt{s}$; and ſet the declination circle to the complement of the ſame angle, to 90°.—By means of the equatorial and azimuth circles moved in their own planes direct the line of collimation to the ſtar :—obſerve the arc intercepted between the equatorial index and XII.—the ſine of half this arc, multiplied into $\sqrt{3}$, (rad. being = 1) will be the ſine of half the zenith diſtance required.

OF PARALLAX, AND THE DETERMINATION OF INACCESSIBLE DISTANCES.

1. Let a line be drawn perpendicular to the diſtance between an adjacent object and any given ſtation, the apparent places of the object, when viewed from the extremities of the line, will be different.

1. The perpendicular line above deſcribed, is called a baſe.

2. The extremities of the baſe, are called ſtations.

3. The angle ſubtended by the extremities of the baſe at the object is called the angle of parallax.

4. The baſe is to the leſſer of the two diſtances of the object from the extremities of the baſe, as the tangent of parallax to radius; and to the greater, as the ſine of the ſame angle to radius.

2. Suppoſe lines to be drawn from the two ſtations to an object, one of the angles contained by theſe lines, and the baſe being a right angle, the other will be the complement of the parallax to 90°.

Hence, if the angles at the ſtations terminating a given baſe be known, the parallax, and conſequently the diſtance of the object, may be determined.

3. If the diſtance of an object be greater than 100000 times the baſe, the angles at the ſtations will not ſenſibly differ from two right ones, the lines drawn from the object to the ſtations are therefore, phyſically ſpeaking, parallel.

One of the angles at the baſe, in all the propoſitions contained in this ſection, is ſuppoſed to be 90°.

The moſt accurate inſtruments conſtructed for the menſuration of angles, cannot be depended upon, to leſs than 2″ the tangent of which is to radius as 1 to 103132.

That angle whoſe tangent is to radius as 1 to 100000, is 2″.06, or little more than two ſeconds.

4. The parallax of an object which is above 100000 times more diſtant than ~~either of~~ the two ſtations of obſervation, is inſenſible.

If the object be at a greater diſtance from either ſtation, than 100000 times the baſe, the angle at one of the ſtations being 90°, the angle at the other will be more than 89° 59′, 57″.9; the difference of which angle and 90° being ſcarcely more than 2″, is too ſmall to become ſenſible by obſervation.

5. If the parallax of an object (obſerved with an inſtrument ſufficiently accurate to meaſure an angle of 2″) be inſenſible, the diſtance of it from either ſtation, cannot be leſs than 100000 times the baſe, from the extremities of which it is obſerved.

It is to be remarked, that, although the diſtance of the object cannot be leſs than 100000 times the baſe, yet it may be greater, in any aſſignable ratio.

Lines drawn from any given points in a baſe, to an object, may be eſteemed in practice parallel, without ſenſible error, if the diſtance of the object is more than 100000 times the baſe.

Rays diverging from any point of the ſun's diſk, upon the ſurface of the earth, may be eſteemed parallel, if their diſtance from each other exceed not about 1000 miles, at the earth's ſurface: becauſe 1000 miles is to the earth's diſtance from the ſun, in proportion of little more than 1 to 100000.

In the ſame manner, plumb lines hanging freely and at reſt, not being diſtant more than about 68 yards from each other, may be accounted parallel.

Rays diverging from a fixed ſtar, to any parts of the earth's orbit, are, phyſically ſpeaking, parallel, becauſe the parallax of the earth's orbit, ſeen at the ſtar, is leſs than 2″.

Two planes drawn parallel to each other, and paſſing through the extremities of a diameter of the earth's orbit, if produced, will appear to coincide with the ſame great circle of the heavens; becauſe the diameter of the earth's orbit when ſeen from the fixed ſtar, ſubtends an angle leſs than 2″.—In like manner, if a plane, paſſing through the earth's center, be parallel to a plane drawn to the ſurface, theſe planes, when produced, apparently coincide with the ſame great circle in the heavens.

OF THE SOLAR SYSTEM.

1. THE ſolar ſyſtem conſiſts of the Sun; and the Planets, and Comets revolving about it.

2. There are ſix primary planets; viz. Mercury, Venus, the Earth, Mars, Jupiter, and Saturn, all of which revolve according to the order of the ſigns in nearly circular orbits round the Sun as a center.

The apparent annual revolution of the Sun in the ecliptic ariſes from the real motion of the Earth in its orbit; Venus and Mercury revolve in orbits which are contained within that of the earth, and are called inferior planets; Mars, Jupiter, and Saturn, which revolve in orbits exterior to the earth's orbit, are called ſuperior planets.

The orbits of all the planets are elliptical, but the principal Phenomena which demonſtrate the truth of the Copernican ſyſtem, are the ſame, whether the orbits are conſidered as elliptical or circular. The latter ſuppoſition is uſually adopted in giving a general deſcription of the diſpoſition and motion of the heavenly bodies.

3. The planes of the orbits in which the planets revolve are inclined differently to the plane of the ecliptic, but none ſo much as 8°.

The latitude of no two planets can differ ſo much as 16°.

4. Three of the primary planets have ſatellites or ſecondary planets revolving round them as centers, in orbits nearly circular, and accompanying them in their revolutions round the Sun.

5. The Moon is the only ſatellite attendant upon the earth, and performs its periodic revolution in about 27 days 7 hours, de-

ſcribing an orbit which is inclined to the plane of the ecliptic at an angle of about 5°.

If the periodic times of any two of the heavenly bodies be *a* and *b*, the ſynodical period will $= \frac{ab}{a-b}$, ſuppoſing their orbits to be circular, and their motion uniform.

The periodical time of the Moon $= 27.29$; the periodic time of the earth $= 365.25$ days; wherefore a ſynodical month $= \frac{365.25 \times 27.29}{365.25 - 27.29} = 29.49$ or about 29 days and a half.

6. The Moon is obſerved to turn the ſame face towards the Earth in every part of her revolution.

From hence it is inferred, that the Moon turns round her axis, which is nearly perpendicular to the plane of her orbit, in the ſame time as ſhe compleats her revolution round the earth.

7. Four ſatellites revolve round Jupiter, and five round Saturn.

8. Saturn is encompaſſed with a luminous ring, the appearance of which varies according to the different poſitions of the ſpectator.

9. The planets and their ſecondaries are opake and ſpherical bodies.

10. The earth turns round its axis which is inclined to the plane of the ecliptic at an angle of 66° 32′, compleating its revolution in $23^{h}.\,56^{m}.\,4^{s}$.

The apparent diurnal revolution of the Sun and Stars from eaſt to weſt, ariſes from the real revolution of the earth round its axis from weſt to eaſt.

If the Sun's place in the ecliptic were always the ſame, 360° of the equator would paſs under a given meridian between the time of the Sun's leaving and returning to the ſame meridian. But ſince the mean increaſe of the Sun's right aſcenſion in 24 hours is 59′. 8″, it follows, that during 24 hours, or a mean Solar day, 360°. 59′. 8″ of the equator will paſs under a given meridian: whence we derive the time of the Earth's revolution round its axis by the following proportion; as $360°.\,59'.\,8'' : 360°$ ſo is $\overset{h}{24}$ to $\overset{h}{23}.\overset{m}{56}.\overset{s}{4}$ the time ſought.

The great circle of the heavens, in the plane of which the Earth performs its diurnal revolution, is called the equator.

The different ſeaſons of the year are cauſed by the inclination of the ecliptic to the equator.

11. Other planets have been obſerved to revolve round their axes.

The diurnal revolution of Jupiter, Mars and Venus, have been diſcovered from the ſpots which are viſible upon their diſks; any one of theſe ſpots being obſerved appears to revolve, returning to the ſame poſition which it has departed from at equal intervals of time.

It is diſcovered from the ſpots on the diſk, that the Sun turns round his axis in about 25 days.

12. The Earth is ſurrounded with a thin tranſparent fluid, called collectively the atmoſphere.

The atmoſphere is the cauſe of the twilight, the apparent elevation of the Stars above their true places, and the general diffuſion of the Sun's rays.

13. Comets are a ſort of planets which revolve in very excentric ellipſes, round the Sun which is ſituated in one of the foci.

Comets are obſerved to revolve, both according to the order of the ſigns, and in the contrary direction.

The planes of their orbits are inclined to the plane of the ecliptic at various angles, not being confined within any limit.

PHENOMENA WHICH DEMONSTRATE THE TRUTH OF THE COPERNICAN SYSTEM, AND SOME COROLLARIES.

1. VENUS and Mercury are never ſeen in oppoſition to the Sun.

A planet is ſaid to be in oppoſition, when the Sun and planet's longitudes differ by 180°.

2. The angle of elongation of an inferior planet from the Sun, never exceeds a certain limit.

3. The ſuperior planets are obſerved to be ſometimes in oppoſition, and ſometimes in conjunction.

4. The ſuperior planets are retrograde in oppoſition, and progreſſive in conjunction.

5. There is a certain angle of elongation from the Sun at which a planet appears ſtationary when ſeen from the Earth.

Let the periodic time of a ſuperior planet, be to that of an inferior, as $1 : t$, and let their diſtances from the ſun be as $1 : s$. — an angle, the coſine of which $= \sqrt{\frac{1-s^2}{1-t^2}}$ radius being 1, will be the angle of elongation from the ſun, at which the inferior planet appears ſtationary when obſerved from the ſuperior. —— The angle of elongation of a ſuperior planet, when ſeen ſtationary from an inferior, is obtuſe: and the ſine of it $= \sqrt{\frac{s^2-t^2}{1-t^2}} \times \frac{1}{s}$ radius being 1.

It is here ſuppoſed, that the planets revolve uniformly in the ſame planes, and in circles, the common center of which coincides with the center of the ſun.

The obſervation of the planets ſtations will therefore deviate ſomething from the preceding rules.

6. A ſpectator on the earth ſees that part of a planet's diſk only which is contained between the circles of viſion and illumination.

It is from hence that the moon and the inferior planets appear horned when they are ſituated between the Earth and Sun.

The Circles of viſion and illumination always ſo nearly coincide in the planets Jupiter and Saturn, that theſe planets appear in all ſituations to ſhine with a full diſk.

In Mars, when near Quadrature, theſe Circles being inclined to each other, cauſe a part of the illuminated diſk to be intercepted from our view; in this poſition Mars is ſaid to be gibbous.

7. The parallax of the Sun is about 8″ ½.

This is the parallax of the Sun when viewed in the Zenith and in the Horizon, or the Horizontal parallax. Wherefore the ſemidiamater of the Earth is to the diſtance of the Earth from the Sun as the ſine of 8″ ½ to radius, or as 1 : 24266 ∴ and ſince the radius of the Earth = 3970 miles, the diſtance of the Sun from the earth = 24266 × 3970 = near 97000000 miles.

8. The parallax of a fixed Star ſituated in or near the pole of the ecliptic is not ſenſible.

The parallax is therefore leſs 2″, and conſequently the diſtance of the Star greater than 100000 times the baſe from the extremities of which it is obſerved, that is, greater than 100000 times the diameter of the Earth's orbit, or greater than 100000 × 194000000 miles.

9. The parallax of a fixed ſtar being not more than 2″, the Sun when viewed from the ſtar would appear under an angle leſs than $\frac{32'.6''}{200000}$, or leſs than the hundredth part of a ſecond, and therefore could not be diſtinguiſhed from a point.

Since bodies equal in magnitude and ſplendor to the Sun, being placed at the diſtance of the fixed Stars, would appear to us as the Stars now do, it may be readily inferred, that the fixed Stars are bodies every way ſimilar to the Sun, which is the center of the ſolar ſyſtem.

This being the caſe, the reaſon will appear why a fixed ſtar, when viewed through a teleſcope magnifying two hundred times, appears no other than a point. For the apparent diameter of the ſtar being leſs than $\frac{1}{100}$ part of a ſecond, when magnified two hundred times, will ſubtend an angle of not ſo much as 2″ at the eye of a ſpectator obſerving it in the teleſcope.

The parallax of the fixed ſtar when viewed from the oppoſite parts of the earth's orbit, is here aſſumed 2″; but it is probable that the parallax of the neareſt ſtar, is much leſs, and conſequently the diſtance greater in the ſame proportion, as the parallax is leſs.

10. The appearance of the fixed ſtars in the heavens will be in every reſpect the ſame, in whatever part of the earth's, or even Saturn's orbit, the ſpectator is ſituated.

11. The change of place or parallax of the moon, when ſeen in the zenith and the horizon, is at a medium, 57′.

12. Comets have no ſenſible diurnal parallax.

It follows from hence, that the diſtance of comets muſt be greater than that of the moon from the earth.

13. Comets are obſerved to change their places in the heavens, on account of the earth's motion in its orbit.

It is inferred from hence, that comets, when viſible, are either within the regions of the ſolar ſyſtem, or not very far diſtant from the orbit of Saturn.

CONCERNING ECLIPSES.

1. THE planets and their ſecondaries ſhine by the light of the ſun reflected from them.

2. An opake body intervening between a planet and the ſun, will intercept part, or even all the rays of light from arriving at the planet, according to the magnitude, and proximity of the opake body.

Any ſphere ſubtending an angle at the ſpectator's eye, equal to, or greater than the apparent diameter of the ſun, will totally intercept the rays of light, if the center of it be in a line joining the center of the ſun and the eye of the ſpectator.

3. The ſhadow of the earth is of a determined length, and will totally eclipſe the ſun from any part of the moon's ſurface ſituated in it.

The figure of the earth's ſhadow is a cone; the axis of which is a continuation of the line, which joins the centers of the earth and ſun; the earth being between the ſun and the vertex of the ſhadow.

The length of a planet's ſhadow is to the ſemi-diameter of the planet, as radius to the ſine of the ſun's apparent ſemi-diameter.

4. Suppoſe the poſition of the ſhadow to be reverſed, ſo that the vertex of the cone may be between the ſun and the planet, the axis remaining in the ſame direction as before; a continuation of this cone increaſing, ſine limite, is called the penumbra.

5. If a ſpectator be ſituated in the penumbra of a planet, a part of the ſun's light will be intercepted from his view.

6. The ſun is never eclipſed but at the time of a new moon, and the moon is never eclipſed but when at the full.

7. Eclipſes of the moon and ſun would happen at every full and new moon, if the plane of the moon's orbits were coincident with the plane of the ecliptic.

8. The plane of the moon's orbit is inclined to that of the ecliptic, at an angle of about 5°.

The line in which theſe planes interſect each other, is called the line of the nodes.

9. If the angular diſtance of the moon's center from the node at the time of the new moon, be leſs than about ſixteen degrees and a half, there will be an eclipſe of the ſun.

10. If the angular diſtance of the full moon's center from the node, be leſs than about twelve degrees and a half, the moon will be eclipſed.

The limits of ſolar, being greater than thoſe of lunar eclipſes, there will happen in a given time a greater number of eclipſes of the ſun than of the moon. But an eclipſe of the moon is viſible to the inhabitants of one half of the globe; whereas a ſolar eclipſe can be obſerved on a ſmall proportion of the earth's ſurface only: For this reaſon, at a given place, more lunar eclipſes will be viſible in a certain time, than eclipſes of the ſun.

11. No primary planet can fall into the ſhadow of another primary planet; but part of a planet's ſurface may be eclipſed by falling into the ſhadow of its ſecondary.

Any planet may fall into the penumbra of an inferior planet.

12. When the earth falls into the penumbra of Venus or Mercury, the rays of the ſun are in part intercepted from our view; and the planet in paſſing over the ſun, appears as a dark ſpot upon the diſk.

13. Mercury and Venus would be obſerved to paſs over the diſk of the ſun at every inferior conjunction, if the planes of their orbits coincided with that of the ecliptic.

OF THE PHENOMENA WHICH ARISE FROM THE EXCENTRICITY OF THE EARTH'S ORBIT, AND CONCERNING THE DIVISION OF TIME.

1. **THE apparent diameter and hourly motion of the Sun are obſerved to vary as the earth revolves in its orbit: and the variations are ſuch as demonſtrate the orbit to be elliptical.**

If the earth's orbit be ſuppoſed an excentric circle according to the idea of ſome ancient aſtronomers, deductions formed from this Hypotheſis diſagree with obſervation.

If the earth be ſuppoſed to revolve in an ellipſe round the Sun placed in one of the Foci, whatever deductions are formed from this ſuppoſition, preciſely agree with Phenomena.

2. **Suppoſing the excentricity and ſemiaxis major of the Earth's orbit to be known, if the Sun's place in the ecliptic be given, the apparent ſemidiameter of the Sun may be determined.**

In any ellipſe let $t =$ the ſemiaxis major
$c =$ the ſemiaxis minor
$m =$ the excentricity

Let a point be aſſumed in the circumference of the ellipſe, and let a line be drawn joining the point aſſumed and the focus. Let $l =$ the coſine of the angle contained between the line drawn as above and the axis major, then the diſtance of the aſſumed point from the focus $= \frac{c^2}{t \pm lm}$

It is immediately inferred from hence, that if d'' be the apparent ſemidiameter of the Sun when at its mean diſtance from the earth, in any other poſition in which the coſine of the Sun's longitude from the apogee $= l$, the apparent ſemidiameter will $= d''t \times \frac{t \pm lm}{c^2}$.

It appears that the obſervation of the Sun's apparent ſemidiameter agrees accurately with the preceding deduction, formed a priori from the elliptic hypotheſis, of which this agreement is a confirmation.

In order to illuſtrate this by an example, let it be required to determine the apparent ſemidiameter of the Sun, when his longitude $=$ 71°. 25′. 40″ in the year 1776, ſuppoſing the excentricity of the earth's orbit to be 16, 83, the ſemiaxis major 1000, and the Sun's mean ſemidiameter 16′. 2.″8.

When the Sun's longitude $=$ 71°. 25′. 40″, his angular diſtance from the apogee $=$ 99°. 16′ $-$ 71°. 25′. 40″ $=$ 27°. 50′. 20″: let the coſine of 27°. 50′. 20″ $= l$. From the preceding data and the rule above deſcribed,

ſcribed, we deduce the apparent ſemidiameter of the Sun $= 1000 \times 962.''8.$ $\times \frac{1000 - \cos.\ 27^\circ.\ 50'.\ 20'' \times 16.83.}{999716} = 15'.48''.74.$

It appears alſo, that d'' being the hourly apparent motion of the Sun in the ecliptic when at the mean diſtance, the hourly motion when the Sun is in any point of the ecliptic, the coſine of the longitude from the apogee being l, will $= d' t^2 \times \frac{\overline{t \pm l m}^2}{c^4}$.

Theſe concluſions, and any other derived in a ſimilar manner for the apparent ſemidiameter, and hourly motion of the Sun at a given longitude, do not differ ſenſibly from obſervation, which is (among others) a demonſtration that the orbit in which the earth revolves is an ellipſe, the Sun being in one of the foci.

3. A line joining the earth and ſun deſcribes equal areas in equal times during the revolution of the earth in its orbit.

Since the interſection of the ecliptic and the equator divides the area of the Earth's orbit unequally, it follows that the times of the Sun's entering the vernal and autumnal equinoxes, will divide the year unequally.

The daily increaſe of the Sun's right aſcenſion is variable on two accounts. 1. The variation of the Sun's angular velocity in the ecliptic. 2. The inclination of the ecliptic to the equator. Hence ariſes the inequality of the ſolar days; a ſolar day is the time elapſed between the Sun's leaving and returning to a given meridian; the length of it may be thus determined: Let $t =$ the time of the earth's revolution round its axis; a the increaſe of the Sun's right aſcenſion during 24 hours: we have the following proportion $360^\circ : 360^\circ + a :: t : t \times \frac{360 + a}{360}$ = to the length of the ſolar day required. Or thus: —— Let $a =$ the increaſe of the Sun's right aſcenſion during 24 hours: the length of the ſolar day will be determined by the following proportion, as $360^\circ.\ 59'.\ 8''$ to $360^\circ + a$ ſo is $\overset{h}{24}$ to $\overset{h}{24} \times \frac{360^\circ + a}{360^\circ.\ 59'.\ 8''}$ the length of the ſolar day required.

Cor. 1. Hence it follows, that the ſolar and mean days will never be equal except when the increaſe of the Sun's right aſcenſion during 24 hours is $= 59'.\ 8''$.

Cor. 2. It appears from hence, that the apparent time of day, deduced from the menſuration of the angle contained between the meridian and a circle of declination paſſing through the Sun's center, is moſt commonly different from true or equated time, ſuch as is ſhewn by a well regulated Clock.

The difference between true and apparent time, is called the equation of time; this is previouſly calculated in aſtronomical Ephemerides, for each day in the year. If we determine apparent time by obſervation of the Sun or Stars, true time will be obtained by adding or ſubtracting the equation, correſponding to the time of obſervation.

4. The tropical year is the time elapſed between the Sun's leaving either of the tropics, and returning to the ſame.

The tropical year conſiſts of 365 days, 5 hours, and about 49 minutes.

5. The ſidereal year is the time elapſed between the ſun's leaving any fixed ſtar in the ecliptic, and returning to the ſame ſtar.

The ſidereal year conſiſts of 365 days, 6 hours, and about 9 minutes.

From this difference between the tropical and ſidereal years, it is inferred that the equinoxial points change their places in the heavens, revolving in a direction contrary to the order of the ſigns at the rate of about 1° in 72 years. Hence the Colures will be ſubject to the ſame motion as the equinoxial points, and the pole of the equator will deſcribe a leſſer circle round the pole of the ecliptic at 23° 28′ from it, and will compleat its revolution in 72 × 360 = 25920 years.

The variation of the equator's inclination to the ecliptic is not here conſidered.

6. The ſolar year is not compoſed of any exact number of days.

Hence ariſes the neceſſity of applying a frequent correction to the Calendar, leſt the ſeaſons of the year ſhould ceaſe to correſpond with the ſun's place in the ecliptic, reckoning from the equinoxes or tropics.

The tropical year conſiſts of [illegible] days, 5 hours, and 49 minutes; if therefore it were accounted only 365 days, according to the cuſtom of the Egyptians, the deficient 5 hours and 49 minutes, would in 4 years accumulate to a difference of 23 hours, 16 minutes, by which the calendar would precede the ſun; but if in order to compenſate for this a whole day be inſerted into the calendar every 4th year, this will be allowing too much by 44 minutes, which renders it neceſſary to leave out a day about every hundredth year.

365

CONCERNING THE PROGRESSIVE MOTION OF LIGHT.

1. WHEN Jupiter's ſatelites emerge from the ſhadow of their primary, the light reflected from them after their obſcuration, is obſerved to arrive at their earth, about ſixteen minutes ſooner when Jupiter is in oppoſition, than when he is near conjunction.

Hence it is inferred, that a ray of light paſſes over the earth's orbit in about ſixteen minutes, or nine hundred and ſixty ſeconds of time, and conſequently moves at the rate of $\frac{194000000}{960}$, or about 202083 miles in a ſecond.

2. The fixed Stars appear to deſcribe ellipſes round their true places as centers.

This apparent motion in the fixed ſtars ariſes from the progreſſive motion of light, and the motion of the earth in its orbit.

The longer axes of the ellipſes are parallel to the plane of the ecliptic, and ſubtend angles which are all equal to about $40''$, and the ſhorter axes $= 40'' \times$ *ſine ſtar's latitude.*

The apparent latitude of a given ſtar, is leaſt when the ſun is 90° before the ſtar in longitude.

The apparent longitude is greateſt when the ſtar and ſun differ in longitude by 180°, and is the leaſt when the longitudes of the ſun and the ſtar are the ſame.

3. The velocity of light is to the velocity of the earth in its orbit, as radius is to the tangent of $20''$, or as 10313 to 1.

This proportion is deduced from Dr. Bradley's theory of the ſtars aberration; from the eclipſes of Jupiter's ſatellites, it is inferred, that the velocity of light is to that of the earth in its orbit, as 202083 to 19.31, or as 10465 to 1: the near agreement of theſe proportions, is a confirmation of the principle aſſumed for the explication of the phenomena.

CONCERNING THE CAUSES OF THE PLANETS MOTION.

1. THE primary planets gravitate towards the ſun, and the ſecondary planets towards their reſpective primaries.

A projectile force having been impreſſed upon a planet when at reſt, would cauſe it to proceed uniformly in a right line for ever, unleſs the ſun, by the continual action of its attractive power, deflected the planet from its rectilinear direction. This attraction is called a centripetal force.

The centripetal force varies with the diſtance of the planets from the ſun, ſo that a planet at half the diſtance would be attracted by a quadruple force; at one third of the diſtance, the force of attraction would be nine times as great.

2. The planets revolve in ellipſes very little differing from circles, round the common center of gravity of the planets and the ſun.

The ſun ſo much exceeds the planets in quantity of matter, that the center of gravity of the ſun, and all the planets, however ſituated, is very near the ſun itſelf.

The areas deſcribed by the line joining a planet, and the point to which the centripetal force is always directed, are proportional to the time of a planet's motion in its orbit.

Some of the properties of central forces may be illuſtrated experimentally.

3. Suppoſe two bodies, urged by centripetal and projectile forces, to deſcribe circles round a center of force ſituated in the common center of the circles; if the periodic times be the ſame, the centripetal or centrifugal forces will be in the ſame ratio as the radii of the circles.

4. If the periodic times be directly as the radii, the forces will be inverſely as the radii.

5. If the ſquares of the periodic time be as the cubes of the diſtances, the centripetal forces will be inverſely as the ſquares of the diſtances.

This caſe obtains in the revolutions of the planets and their ſecondaries, this difference excepted, the planets and their ſecondaries perform their motions in ellipſes; whereas in the experiment, the bodies revolve in circular orbits; but the ſame properties are demonſtrated (as far as regards the experiment) when the orbits are ellipſes as in the above example, changing the radius of the circle for the greater ſemiaxis of the ellipſe.

6. The ſpher~~io~~oidical figure of the earth is cauſed by the revolution of the earth round its axis.

The earth is an oblate ſpheroid, an equatorial diameter, being greater than the polar, in the proportion of 230 : 229.

From this figure of the earth, the inclination of the ecliptic to the equator, and the attraction ofthe ſun and moon, ariſes the preceſſion of the equinoxes.

7. The centrifugal force of the earth revolving round the common center of gravity of the earth and moon, contributes to increaſe the tides in thoſe parts of the earth which are moſt remote from the moon.

8. The lunar irregularities are cauſed by the action of the ſun upon the earth and moon.

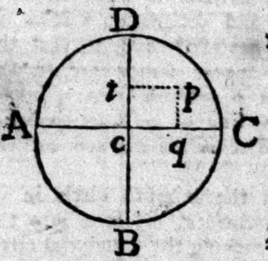
ERRATA.

Page 3. line 25, *for* counterpoize, *read* counterpoiſe.—P. 17. l. 2, *read* perpendicular to the horizon. — P. 21. l. 2. *for* up an immeſed tube, *read* up an immerſed tube. — P. 25. l. 19. *for* cubic in inch, *read* cubic inch. — P. 27. l. 7. *for* ſpeciefic, *read* ſpecific. — l. 20, at the end, *dele* or a little. — P. 59. l. 6. *for* on the oppoſite direction, *read* in the oppoſite direction. — P. 60. l. 21. after nearly, *inſert* (:). — P. 63. l. 22. *dele* be. — P. 66. laſt line, *dele* upon the retina. — P. 68. l. 24. *for* there, *read* theſe. — P. 22. l. 21. *for* object, *read* body. — P. 73. l. 6. *for* is obſerved, *read* are obſerved. — P. 91. l. 14. *for* $n \pm 1$, *read* $n+1$.—P. 104. l. 37. *read* equatorial. — P. 104. l. 39. *read* apparent places. — P. 90. l. 39. *for* verticle, *read* vertical. — P. 110. l. 5. *dele* and. — P. 92. l. 35. *for* direction, *read* obſervation. — P. 91. l. 13. *read* d'' and o, is equal, &c. l. 26. *read* combinations. — P. 101. l. 3. *read* revolve in a leſſer circle. — P. 108. l. 8. *dele* either of. — P. 112. l. 7. *for* planets, *read* planet.

APPENDIX.

A METHOD OF ADJUSTING THE EQUATORIAL INSTRUMENT.

Fig. I.



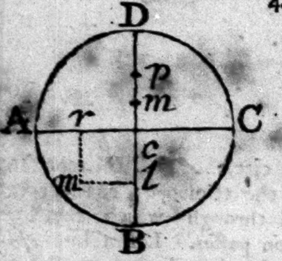
1. LET the circle $ABCD$ repreſent the field of the teleſcope: c, the point in the field through which the line of collimation paſſes. Let a line AC paſſing through c be adjuſted parallel to the plane of the declination circle, and ſuppoſe BD to be drawn through c perpendicular to AC.

2. Having ſet the declination circle to 90° (in either quadrant) direct the point c to a very diſtant fixed object p, and turn the equatorial circle in its own plane 180°.---If p does not now coincide with the point c, imagine the lines pq, pt to be drawn perpendicular to AC and DB reſpectively: correct half the error cq by moving the declination circle in its own plane, and half the error pq, by altering the inclination of the declination circle to the plane of the equatorial.---If the point c be now directed to the diſtant object p, and the equatorial circle be turned 180° in its own plane, the points p and c will ſtill coincide.—By a very diſtant object is meant any viſible point, the diſtance of which is not leſs than 400000 times the perpendicular diſtance of the line of collimation from the axis of the declination circle.

3. Set the declination nonius to 90; and having turned the declination circle 180° in its own plane, direct the point c to the diſtant object p; *(Fig. 1.)* and move the equatorial circle in its own plane 180°.—If p now deviates from c, the error cq will ſhew double the error in the diviſion of the declination circle, and cannot be corrected by adjuſtment: ſuppoſe therefore the point p to be ſomewhere in the line DB, and correct $\frac{1}{4}$ of the error tc by altering the inclination of the declination circle to the plane of the equatorial, and $\frac{1}{4}$ of the ſame error tc by moving the line of collimation perpendicular to the plane of the declination circle.

When the preceding adjuſtments are correct it appears, 1. That the line of collimation is parallel to the plane of the declination circle. 2. That the plane of the declination circle is perpendicular to that of the equatorial. 3. The errors of diviſion, (if any) in the two oppoſite points of the declination circle (*viz.* the two 90°s) are diſcovered. 4. When the declination circle is ſet to 0, the line of collimation is parallel, and when ſet to 90°, (in either quadrant) is perpendicular to the plane of the equatorial circle.—

Fig. II.



4. The azimuth circle being adjuſted horizontal, make A C parallel to it by turning the equatorial and azimuth circles in their own planes, and having directed the point c to a very diſtant fixed object p, eſtimated to be in the horizon or near it, move the azimuth and declination circles in their own planes 180° each, ſo that the point p may appear ſomewhere in the line DB: if p deviates from c, biſſect $p\,c$ in m, the line of collimation being directed to m, will be horizontal.—Let m be marked on the horizon.
5. Set the declination circle to 0, and turn the azimuth circle in its own plane 90°, ſo as to bring the horizontal object m into the line D B, and direct the point c to m, by moving the equatorial circle in its own plane if neceſſary.—The plane of the declination circle will now be horizontal, and that of the equatorial circle vertical.
6. Turn the equatorial and azimuth circles in their own planes 180°. If m now deviates from c, $m\,r$ will be the error in diviſion of the equatorial, and $m\,l$ the error of diviſion in the azimuth circle. (*Fig. II.*)
7. In order to elevate the equatorial circle, above the horizon to any propoſed inclination, ſet the declination circle (in the ſouthern quadrant) to the propoſed angle of elevation, and having brought XII on the equatorial circle to point 0 on the index, direct the line of collimation to the horizontal point m, by altering the inclination of the equatorial circle to the horizon, and by turning the azimuth circle in its own plane.
8. In order to adjuſt the equatorial index, ſet the declination circle to 0, and by moving the equatorial and azimuth circles in their own planes, direct the line of collimation to the horizontal point m, and ſet the equatorial index to VI.
9. By means of the laſt adjuſtment, when the equatorial index points to XII. the plane of the declination circle will coincide with a ſecondary common to the equatorial and azimuth circles; upon this ſecondary therefore the inclination of thoſe circles to each other will be truely meaſured.—The 7th adjuſtment may be repeated if it be thought neceſſary.
10. If the teleſcope magnifies 60 times, it follows from the preceding adjuſtments, that any inclination of the line of collimation, or of the equatorial axis to the plane of the declination circle muſt be diſcovered, if it exceeds 1″;—It follows alſo that, (if the levels by means of which the poſition of the azimuth circle is rectified be tolerably good) the point m (deſcribed in art. 4) is truly horizontal within 6″ or 8″.——

It may be inferred from hence, that when the plane of the equatorial circle coincides with that of the equator on the heavens, the line of collimation will during the motion of the declination circle turning in its own plane, paſs through the pole of the equator, without ſenſible error, and will conſequently deſcribe a circle of declination in the heavens: when the equatorial index points to XII, the line above mentioned will deſcribe a ſecondary common to the equator and horizon, or the meridian of the place, and will therefore paſs through the pole of the equator and the zenith.---The equatorial inſtrument in this poſition may be uſed as a tranſit teleſcope..

The errors in the poſition of the meridian, and in the time deduced from obſervation with the equatorial inſtrument (deſcribed and eſtimated in page 105.) may be corrected thus.---The teleſcope being of that ſort which repreſents the image inverted with reſpect to the object, it follows that the ſtar's apparent elevation ariſing from refraction will cauſe the image in the teleſcope to be depreſſed through a ſpace which ſubtends an angle at the center of the object glaſs, equal to the ſtar's refraction in altitude.---Suppoſe DB (*Fig. II.*) to be perpendicular to the line of collimation, and to be adjuſted to the plane of a vertical circle which paſſes through an obſerved ſtar: let *c l* repreſent the depreſſion of the ſtar's image above deſcribed, which may be determined, if the principal focal length of the object glaſs, and the ſtar's reſtaction in altitude be given: if *l* be now directed to the ſtar's apparent place, it is manifeſt, that the line of collimation of the inſtrument which joins the point *c* and the center of the object glaſs, will be directed to the true place of the ſtar, whereby the effects of refraction upon the hour angle and poſition of the meridian are intirely removed.

In determining the right aſcenſions and declinations of comets or ſtars by an equatorial inſtrument, perfectly adjuſted to the latitude and meridian, conſiderable errors ariſe from refraction.---Let $\dot{r}$ repreſent the refraction in altitude, a the right aſcenſion, and d the declination of a ſtar; $\dot{a}$ and $\dot{d}$ the variations of right aſcenſion and declination cauſed by refraction. Alſo, let s = the angle contained between a vertical circle, and circle of declination paſſing through the ſtar: we have from the properties of ſpherics $\dot{a} = \dot{r} \times \frac{\text{ſin. s}}{\text{Coſ.}\, d}$; and $\dot{d} = \dot{r} \times \text{Coſ. s}$; radius being 1. Theſe errors in determining right aſcenſions and declinations with the equatorial inſtrument are (as in the former caſe) corrected in the obſervation, if the point *c* through which the line of collimation paſſes, be previouſly depreſſed in a vertical plane through a ſpace *c l* which ſubtends an angle at the center of the object glaſs equal to the ſtar's refraction in altitude.

This is one among many other ſcientific improvements which have been applied to the conſtruction of the equatorial inſtrument by Mr. Ramsden.

Two formulæ for the conſtruction of perfect lenſes or object glaſſes, vide Euler. Diopt. page 335. vol. 1.

Theſe object glaſſes conſiſt of three contiguous lenſes, having a common axis.—The two outward lenſes are double convex, made of crown glaſs, encloſing a double concave of flint glaſs.

The ratio of refraction, in the rays of mean refrangibility paſſing out of air into crown glaſs, is that of - - 1.53 : 1
Out of air into flint glaſs, as - - - 1.58 : 1
The diſſipating powers of flint and crown glaſs are as 3 : 2
The principal focal length of the perfect lenſes = P.

FIRST FORM.

The radii of the ſurfaces are as follows:

$$\text{Radii of the} \left\{ \begin{array}{l} \text{1ſt ſurface} = 0.5004 \times P \\ \text{2d ſurface} = 3.6665 \times P \end{array} \right\} \text{crown glaſs.}$$

$$\text{Radii of the} \left\{ \begin{array}{l} \text{1ſt ſurface} = -\,0.5167 \times P \\ \text{2d ſurface} = -\,0.4843 \times P \end{array} \right\} \text{flint glaſs.}$$

$$\text{Radii of the} \left\{ \begin{array}{l} \text{1ſt ſurface} = 0.5219 \times P \\ \text{2d ſurface} = 0.4757 \times P \end{array} \right\} \text{crown glaſs.}$$

$$\text{Semiaperture} = 0.1189 \times P$$

SECOND FORM.

$$\text{Radii of the} \left\{ \begin{array}{l} \text{1ſt ſurface} = 0.2829 \times P \\ \text{2d ſurface} = 2.0729 \times P \end{array} \right\} \text{crown glaſs}$$

$$\text{Radii of the} \left\{ \begin{array}{l} \text{1ſt ſurface} = -\,2.1459 \times P \\ \text{2d ſurface} = -\,0.2955 \times P \end{array} \right\} \text{flint glaſs.}$$

$$\text{Radii of the} \left\{ \begin{array}{l} \text{1ſt ſurface} = 0.5938 \times P \\ \text{2d ſurface} = 2.5006 \times P \end{array} \right\} \text{crown glaſs.}$$

$$\text{Semiaperture} = 0.0707 \times P.$$

N. B. The firſt ſurface is ſuppoſed to be that which is neareſt to the object.

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